# Workshop 7: Forced and natural convection

Attempt these problems before opening the worked solutions.

## 1 Water cooling a reactor tube

EXERCISE
**Exercise 1. Uniform heat flux**

Water at $16\,^{\circ}\mathrm{C}$ enters a 2.54 cm ID tube that is used to cool a nuclear reactor. The water flow rate is $0.114\ \mathrm{m}^3/\mathrm{min}$. Using the Colburn analogy, determine the total rate of heat transfer, the exiting water temperature and the wall temperature at the exit, if the length of the tube is 4.5 m and the wall condition is one of uniform heat flux of $5.7 \times 10^3\ \mathrm{kJ/hr.m^2}$.

See solution for Exercise 1

## 2 Oil in a cold lake

EXERCISE
**Exercise 2. Thermally developing laminar flow**

Consider the flow of oil in a 30 cm diameter pipeline at an average velocity of 2 m/s and temperature of $20\,^{\circ}\mathrm{C}$. A section of the pipeline (200 m long) passes through icy waters of a lake at $0\,^{\circ}\mathrm{C}$. Measurements indicate that the surface temperature of the pipe is very nearly $0\,^{\circ}\mathrm{C}$. Disregarding the thermal resistance of the pipe material, determine:

a. The temperature of the oil when the pipe leaves the lake;

b. The rate of heat transfer from the oil.

Properties of oil (@ $20\,^{\circ}\mathrm{C}$): $\rho$ = 888 $\mathrm{kg/m^3}$; k = 0.145 $\mathrm{W/m.K}$; $\nu$ = 901 × $10^{-6}$ $\mathrm{m^2/s}$; $c_p$ = 1880 $\mathrm{J/kg.K}$.

See solution for Exercise 2

## 3 Water pipe in moving or stagnant air

EXERCISE
**Exercise 3. Convection resistances per unit length**

A thick-walled steel pipe (k = 60 $\mathrm{W/m.K}$) carrying hot water is cooled externally by a cross-flow airstream at a velocity of 20 m/s and a temperature of $25\,^{\circ}\mathrm{C}$. The inner and outer diameters of the pipe are 20 mm and 25 mm, respectively. At a certain location

along the pipe, the mean temperature of the water is $80\,^{\circ}\mathrm{C}$. Assuming the flow inside the tube is fully developed with a Reynolds number of 20,000, find the heat transfer rate to the air stream per unit pipe length. What will be the heat transfer rate if the surrounding air is stagnant?

See solution for Exercise 3

# Solutions

## Exercise solutions

### Exercise 1. Uniform heat flux

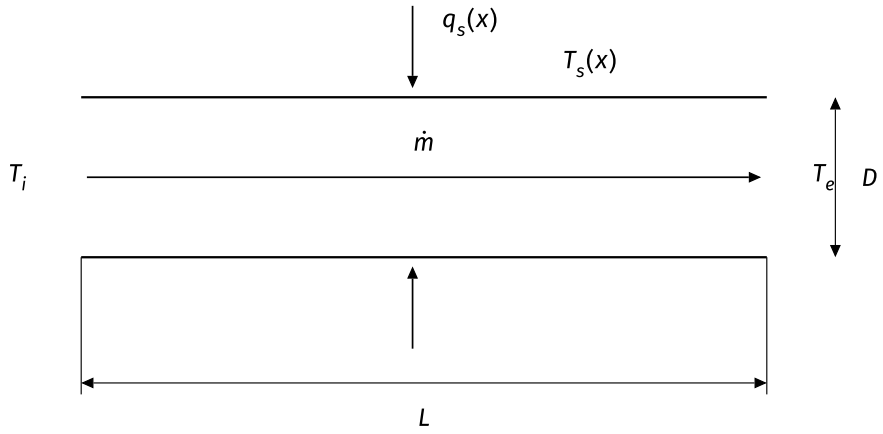



Figure 1: Water flows through the heated tube. The specified inward heat flux supplies the bulk enthalpy rise; the local outlet coefficient determines the exit wall temperature.

**Data**

| Quantity | Symbol | Value |
|---|---|---|
| Tube diameter and length | $D, L$ | $0.0254, 4.5\ \mathrm{m}$ |
| Volume flow | $\dot{V}$ | $0.114\ \mathrm{m^3\ min^{-1}}$ |
| Inlet temperature | $T_{in}$ | $16\ ^\circ\mathrm{C}$ |
| Uniform wall flux | $q''$ | $5.7 \times 10^3\ \mathrm{kJ\ h^{-1}\ m^{-2}}$ |

| Water property | Constant value |
|---|---|
| Density $\rho$ | $1000\ \mathrm{kg\ m^{-3}}$ |
| Specific heat $c_p$ | $4180\ \mathrm{J\ kg^{-1}\ K^{-1}}$ |
| Dynamic viscosity $\mu$ | $0.001\ \mathrm{Pa\ s}$ |
| Prandtl number $Pr$ | $7.0$ |

**Assumptions**

Use the source solution’s rounded water properties throughout. Assume steady single-phase flow, fully developed turbulent heat transfer, negligible axial conduction and no heat loss elsewhere. Ignore entrance effects. The prescribed heat flux acts on the inner wetted surface.

**Calculations**

For this smooth tube use

$$\begin{aligned} Re &= \frac{\rho u D}{\mu}, \qquad f_F = 0.0791 Re^{-1/4}, \\ St\, Pr^{2/3} &= \frac{f_F}{2}, \qquad St = \frac{h}{\rho u c_p}. \end{aligned} \tag{1}$$

Here $f_F$ is the Fanning friction factor, one quarter of the Darcy factor. The correlation applies to smooth turbulent flow in the approximate range $4000 < Re < 10^5$. Use

$$\dot{Q} = \dot{m} c_p (T_{out} - T_{in})$$

and the local wall relation $q'' = h(T_w - T_b)$.

The imposed heat flux fixes the duty before a convection coefficient is needed.

The supplied flux is in kJ per hour per square metre. Convert energy and time together:

$$q'' = 5.7 \times 10^3 \frac{1000}{3600} = 1583.33\ \mathrm{W\,m^{-2}}$$

The wetted area is $\pi DL = 0.359084\ \mathrm{m^2}$, hence

$$\dot{Q} = q'' \pi DL = 568.55\ \mathrm{W}.$$

The flow rate is $0.0019\ \mathrm{m^3\,s^{-1}}$ and $\dot{m} = 1.900\ \mathrm{kg\,s^{-1}}$.

The bulk energy balance gives

$$T_{out} = 16 + \frac{568.55}{1.900(4180)} = 16.0716\ ^\circ\mathrm{C}.$$

The cross-sectional area gives $u = 3.7497\ \mathrm{m\,s^{-1}}$, $Re = 95242$ and $f_F = 0.0045027$.

This is turbulent and within the stated smooth-tube correlation range.

Equation Eq. 1 gives

$$h = \frac{f_F}{2} \rho u c_p Pr^{-2/3} = 9643\ \mathrm{W\,m^{-2}\,K^{-1}},$$

$$T_{w,out} = T_{out} + \frac{q''}{h} = 16.2358\ ^\circ\mathrm{C}.$$

The wall is hotter than the local bulk water, as required for inward heat transfer.

The source's chart estimate $f_F = 0.0046$ gives a slightly different coefficient.

The supplied smooth-tube formula removes chart-reading ambiguity.

**Answer**

The water receives 569 W, exits at 16.07 °C, and has an exit wall temperature of 16.24 °C.

The water temperatures retain two decimal places to show the small temperature rise.

Back to Exercise 1

### Exercise 2. Thermally developing laminar flow

#### Data

| Quantity | Symbol | Value |
|---|---|---|
| Cold-section diameter and length | $D, L$ | $0.30, 200\ \mathrm{m}$ |
| Mean speed | $u$ | $2.0\ \mathrm{m\,s^{-1}}$ |
| Inlet and wall temperatures | $T_{in}, T_s$ | $20, 0\ ^\circ\mathrm{C}$ |
| Density | $\rho$ | $888\ \mathrm{kg\,m^{-3}}$ |
| Conductivity | $k$ | $0.145\ \mathrm{W\,m^{-1}\,K^{-1}}$ |
| Kinematic viscosity | $\nu$ | $901 \times 10^{-6}\ \mathrm{m^2\,s^{-1}}$ |
| Specific heat | $c_p$ | $1880\ \mathrm{J\,kg^{-1}\,K^{-1}}$ |

#### Assumptions

Assume steady hydrodynamically developed flow at the start of the cold section, a uniform inlet bulk temperature, constant properties and negligible viscous heating and axial conduction. Neglect pipe-wall resistance, so the inner wall is also isothermal at $T_s = 0\,^\circ\mathrm{C}$. The thermal boundary layer starts at the lake entrance. Check $Re = uD/\nu$ and $Pr = \rho\nu c_p/k$.

#### Calculations

For laminar flow, estimate the thermal entry length with

$$L_t = 0.0575 Re\, Pr\, D.$$

Use the length-average laminar entrance correlation and integrated bulk balance for constant wall temperature:

$$\begin{aligned} Gz &= Re\, Pr\, \frac{D}{L}, \\ \overline{Nu}_D &= 3.66 + \frac{0.065 Gz}{1 + 0.04 Gz^{2/3}}, \\ \overline{h} &= \frac{\overline{Nu}_D k}{D}, \\ \frac{T_{out} - T_s}{T_{in} - T_s} &= \exp\left(-\frac{\overline{h}\pi D L}{\dot{m} c_p}\right). \end{aligned} \tag{2}$$

The correlation models thermal development with a developed velocity profile; ignore wall-to-bulk viscosity corrections under the stated constant-property assumption. Do not substitute the fully developed value $Nu = 3.66$ prematurely.

The oil velocity profile is laminar but its thermal boundary layer is still developing.

The supplied properties give $Re = 665.93$, $Pr = 10373.55$ and $L_t = 1.1916 \times 10^5\,\mathrm{m}$, far longer than the cold section.

The large Prandtl number explains the long thermal entry region.

Equation Eq. 2 gives $Gz = 10362.04$, $\overline{Nu}_D = 37.3168$ and $\overline{h} = 18.0364\,\mathrm{W\,m^{-2}\,K^{-1}}$.

The mass flow rate is

$$\dot{m} = \rho u \frac{\pi D^2}{4} = 125.538\,\mathrm{kg\,s^{-1}}.$$

The dimensionless exponent compares the wall conductance with the heat-capacity flow rate:

$$\frac{\overline{h}\pi DL}{\dot{m}c_p} = \frac{18.0364\pi(0.30)(200)}{125.538(1880)} = 0.0144052$$

The exponential bulk balance then gives

$$T_{out} = 20\exp(-0.0144052) = 19.7140\,^\circ\mathrm{C},$$

$$\dot{Q}_{loss} = \dot{m}c_p(20 - T_{out}) = 67508\,\mathrm{W} = 67.51\,\mathrm{kW}.$$

The oil cools only 0.286 degrees because the heat-capacity flow rate is large.

This positive heat-loss convention is the magnitude of the negative heat input to the oil.

Retaining intermediate precision avoids the handwritten source's rounded outlet temperature changing the reported duty to about 68.4 kW.

**Answer**

The oil exits at $19.7\,^\circ\mathrm{C}$ and loses $67.5\,\mathrm{kW}$ over the cold section. The unrounded energy balance uses a bulk temperature drop of 0.286 K. The flow is laminar but thermally developing.

Back to Exercise 2

## Exercise 3. Convection resistances per unit length

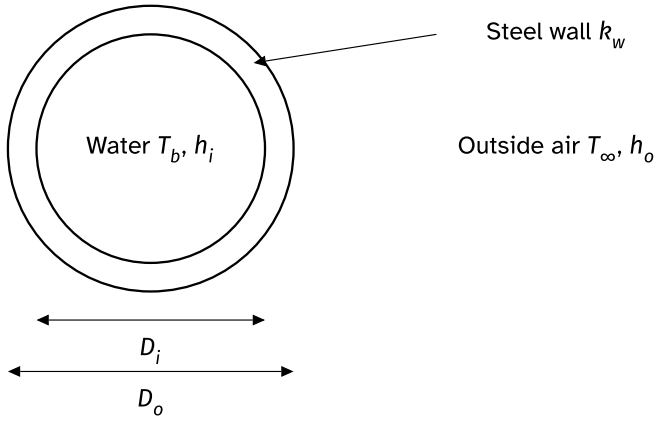



Figure 2: Heat passes from the water through the inside film, steel wall and outside air film. The inside and outside coefficients act on different cylindrical areas.

### Data

| Quantity | Symbol | Value |
|---|---|---|
| Inside and outside diameters | $D_i, D_o$ | $0.020, 0.025\ \mathrm{m}$ |
| Steel conductivity | $k_{steel}$ | $60\ \mathrm{W\,m^{-1}\,K^{-1}}$ |
| Water and ambient air temperatures | $T_b, T_\infty$ | $80, 25\ ^\circ\mathrm{C}$ |
| Internal Reynolds number | $\mathrm{Re}_i$ | 20000 |
| Cross-flow speed | $u_\infty$ | $20\ \mathrm{m\,s^{-1}}$ |

The property approximations are

| Property | Water | Moving air | Stagnant air film |
|---|---|---|---|
| Conductivity $k$, $\mathrm{W\,m^{-1}\,K^{-1}}$ | 0.67 | 0.0263 | 0.0282 |
| Prandtl number $Pr$ | 2.2 | 0.707 | 0.707 |
| Kinematic viscosity $\nu$, $\mathrm{m^2\,s^{-1}}$ | Not needed | $15.89 \times 10^{-6}$ | $18.2 \times 10^{-6}$ |

### Assumptions

- Steady radial wall conduction with negligible fouling and axial wall conduction.
- An isolated long horizontal pipe with radiation excluded.
- The local bulk water temperature is fixed in both air cases.

Water and moving-air values follow the source solution. The stagnant-air values approximate air at 325 K and stay fixed while the film temperature in $\beta = 1/T_f$ is iterated. Use $g = 9.81\,\mathrm{m\,s^{-2}}$.

**Calculations**

For internal water use the source's Colburn form; for cylinder cross-flow use the stated Zukauskas approximation with the surface-Prandtl correction set to one:

$$\begin{aligned} Nu_i &= 0.023 Re_i^{0.8} Pr_i^{1/3}, \\ h_i &= \frac{Nu_i k_i}{D_i}, \\ Re_o &= \frac{u_\infty D_o}{\nu_o}, \\ Nu_o &= 0.26 Re_o^{0.6} Pr_o^{0.37}, \\ h_o &= \frac{Nu_o k_o}{D_o}. \end{aligned}$$

The outside correlation applies to $1000 < Re_o < 2 \times 10^5$ and $Pr_o < 10$. This explicitly uses the source's $1/3$ internal exponent rather than the Dittus-Boelter cooling exponent. Resistances per unit length have units $\mathrm{m\,K\,W^{-1}}$:

$$\begin{aligned} R_i' &= \frac{1}{h_i \pi D_i}, \\ R_w' &= \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi k_{steel}}, \\ R_o' &= \frac{1}{h_o \pi D_o}, \\ q' &= \frac{80 - 25}{R_i' + R_w' + R_o'}. \end{aligned} \tag{3}$$

For stagnant air, the horizontal-cylinder Churchill-Chu relation is

$$Nu_D = \left[ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + \left(\frac{0.559}{Pr}\right)^{9/16}\right]^{8/27}} \right]^2.$$

Use

$$Ra_D = \frac{g\beta(T_{s,o} - 25) D_o^3 Pr}{\nu^2}$$

within the range $Ra_D < 10^{12}$, and recover $h_o = Nu_D k / D_o$. Evaluate the properties at

$$T_f = \frac{T_{s,o} + 25}{2} + 273.15\,\mathrm{K}.$$

The outer surface temperature must satisfy both

$$q' = \frac{80 - T_{s,o}}{R'_i + R'_w}$$

and

$$q' = h_o \pi D_o (T_{s,o} - 25).$$

Solve iteratively.

The external air film controls most of the forced-convection resistance.

The internal calculation gives $Nu_i = 82.5454$ and $h_i = 2765.27\,\mathrm{W\,m^{-2}\,K^{-1}}$.

Outside, $Re_o = 31466.3$ is within the stated range, $Nu_o = 114.279$ and $h_o = 120.221\,\mathrm{W\,m^{-2}\,K^{-1}}$.

Using Eq. 3 gives

$$R'_i = 0.0057555\,\mathrm{m\,K\,W^{-1}},$$

$$R'_w = 0.00059191\,\mathrm{m\,K\,W^{-1}},$$

$$R'_o = 0.105908\,\mathrm{m\,K\,W^{-1}},$$

$$q'_{forced} = 489.95\,\mathrm{W\,m^{-1}}.$$

The outer surface is

$$25 + q' R'_o = 76.8901\,^\circ\mathrm{C}.$$

The handwritten resistance labels transpose the inner and outer coefficients; their sum and approximate 489.5 W per metre result are nevertheless consistent.

Here each resistance uses its correct coefficient and diameter.

For stagnant air, trial an outer surface temperature between 25 and 80 degrees and equate the conductive/internal flux to the natural-convection flux.

The converged values are

$$T_{s,o} = 79.7947\,^\circ\mathrm{C} \qquad T_f = 325.547\,\mathrm{K}$$

$$Ra_D = 55066.9 \qquad Nu_D = 6.66158$$

$$h_o = 7.5143\,\mathrm{W\,m^{-2}\,K^{-1}} \qquad q'_{natural} = 32.338\,\mathrm{W\,m^{-1}}$$

Both heat-rate calculations agree:

$$h_o \pi D_o (T_{s,o} - 25) = \frac{80 - T_{s,o}}{R'_i + R'_w} = 32.338\,\mathrm{W\,m^{-1}}.$$

The natural-convection Rayleigh number lies within the specified range.

The final film is close to the supplied 325 K property approximation.

Stagnant air removes much less heat and leaves the wall closer to the water temperature.

Radiation was excluded, so this is a convection-only comparison.

**Answer**

The heat loss is approximately $490\ \mathrm{W\,m^{-1}}$ in moving air and $32.3\ \mathrm{W\,m^{-1}}$ in stagnant air. The latter is the convection-only result for the specified horizontal pipe.

Back to Exercise 3