Workshop 5: Steady heat conduction

Attempt these problems before opening the worked solutions.

1 Insulated steam pipe

EXERCISE
Exercise 1. Heat loss through an insulated pipe

Saturated steam at $130\,^{\circ}\mathrm{C}$ is flowing inside a steel pipe having an ID of 2.09 cm and an OD of 2.67 cm. The pipe is insulated with 3.8 cm of insulation on the outside. The convective coefficient for the inside steam surface of the pipe is estimated to be h_i = 5680 $\mathrm{W/m^2.K}$, while on the outside of the insulation it is estimated to be h_o = 11.4 $\mathrm{W/m^2.K}$. The mean thermal conductivities of the metal and the insulation material are 45 $\mathrm{W/m.K}$ and 0.064 $\mathrm{W/m.K}$, respectively. Calculate the heat loss for 1 m of pipe using the method of resistances if the surrounding air is at $26\,^{\circ}\mathrm{C}$.

See solution for Exercise 1

2 Electrically heated slab

EXERCISE
Exercise 2. Insulation surface temperature

In a laboratory experiment, an electric current is passed through a slab of thickness 2δ. As a result, heat generation S_e takes place in the slab. The slab is insulated on both sides (thickness = t) to satisfy the safety considerations related to the danger stemming from the fact that an electric current is passing. In addition, the insulated slab is cooled on both sides by a fluid of temperature T_c, with convective coefficient of h. Determine the temperature at the surface of the insulation, assuming that the thermal conductivities for the slab and the insulation material are constants, denoted by k_s and k_i, respectively.

See solution for Exercise 2

3 Pipe location in a hot wall

EXERCISE
Exercise 3. Temperature-dependent conductivity

A pipe designed to transport liquid metal is to be embedded in a 1.2 m thick wall. The inside surface of the wall has a constant surface temperature of 925 K, whereas its external surface is exposed to air with a convective heat transfer coefficient of 23 $\mathrm{W/m^2.K}$ and

300 K. The thermal conductivity of the wall varies with temperature according to $k = 0.073 + 3.942 \times 10^{-4}T$, where T is temperature in Kelvin, and k is in $\mathrm{W/m.K}$. It is desired that the pipe is located at a point in the wall where the temperature is < 650 K. Calculate how far from the hot surface should the pipe be located and the heat flux for the wall.

See solution for Exercise 3

Solutions

Exercise solutions

Exercise 1. Heat loss through an insulated pipe

Heat flows radially from the steam, through the steel and insulation, and then into the air. The same heat rate passes through four resistances.

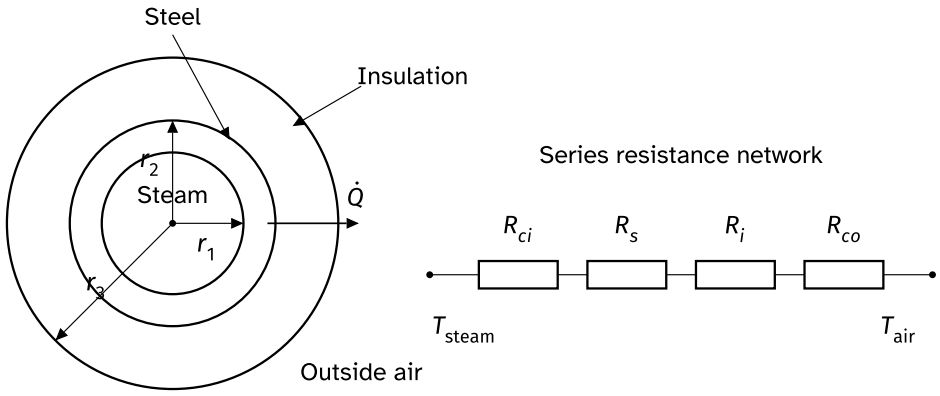


Figure 1: Cross-section of an insulated pipe. The inner and outer steel radii are r1 and r2; insulation extends to r3. Heat crosses the inner film, steel, insulation and outer film in series.

Data

Quantity	Symbol	Value
Pipe length	L	$1.0\ \mathrm{m}$
Inside diameter	D_1	$0.0209\ \mathrm{m}$
Outside steel diameter	D_2	$0.0267\ \mathrm{m}$
Radial insulation thickness	t	$0.0380\ \mathrm{m}$
Steel conductivity	k_s	$45\ \mathrm{W\,m^{-1}\,K^{-1}}$
Insulation conductivity	k_i	$0.064\ \mathrm{W\,m^{-1}\,K^{-1}}$
Steam and air temperatures	T_{steam}, T_{air}	$130, 26\ ^\circ\mathrm{C}$
Inside and outside coefficients	h_i, h_o	$5680, 11.4\ \mathrm{W\,m^{-2}\,K^{-1}}$

We need the heat loss from the specified one-metre length.

Assumptions

- Steady radial heat flow with negligible end losses.
- Constant conductivities and heat transfer coefficients.

- Perfect thermal contact between steel and insulation.
- No separate radiation or fouling resistance.

Calculations

First convert diameters to radii. Insulation thickness is already a radial length, so it is added directly to the outside steel radius:

$$r_1 = \frac{D_1}{2} = 0.01045\ \text{m}, \qquad r_2 = \frac{D_2}{2} = 0.01335\ \text{m}$$

$$r_3 = r_2 + t = 0.01335 + 0.0380 = 0.05135\ \text{m}$$

Each film uses its own cylindrical area, $2\pi rL$. Conduction through a cylindrical layer uses both bounding radii. The four resistances are

Element	Resistance relation	Value, K W^{-1}
Inside steam film	$R_{ci} = 1/(h_i 2\pi r_1 L)$	0.00268
Steel wall	$R_s = \ln(r_2/r_1)/(2\pi k_s L)$	0.000866
Insulation	$R_i = \ln(r_3/r_2)/(2\pi k_i L)$	3.35
Outside air film	$R_{co} = 1/(h_o 2\pi r_3 L)$	0.272

For example, the largest resistance is obtained from

$$R_i = \frac{\ln\left(\frac{0.05135}{0.01335}\right)}{2\pi(0.064)(1.0)} \simeq 3.35\ \text{K W}^{-1}$$

The elements are in series, so

$$R_{total} = R_{ci} + R_s + R_i + R_{co} \simeq 3.626\ \text{K W}^{-1}$$

Use the steam-to-air temperature difference across the complete network:

$$\dot{Q} = \frac{T_{steam} - T_{air}}{R_{total}} = \frac{130 - 26}{3.626} \simeq 28.7\ \text{W}$$

The outer insulation temperature provides a useful direction check:

$$T_{s,o} = T_{air} + \dot{Q} R_{co} \simeq 26 + 28.7(0.272) = 33.8\,^{\circ}\text{C}$$

It lies above the air temperature and below the steam temperature. The insulation supplies about 92 percent of the total resistance. Its large temperature drop is consistent with the small heat loss.

Answer

The heat loss is approximately 28.7 W from one metre of pipe, or $28.7\ \text{W m}^{-1}$. This is a heat rate per pipe length, not a heat flux per surface area.

Back to Exercise 1

Exercise 2. Insulation surface temperature

Choose $x = 0$ at the slab centre and $x = \delta$ at its right face.

Symmetry gives $dT/dx = 0$ at the centre.

The slab energy balance is

$$k_s \frac{d^2T}{dx^2} + S_e = 0.$$

Integrating with the symmetry condition gives

$$\frac{dT}{dx} = -\frac{S_e x}{k_s}.$$

The heat flux leaving either slab face, in $\mathrm{W\,m^{-2}}$, is therefore

$$q'' = -k_s \left[\frac{dT}{dx}\right]_{\delta} = S_e \delta.$$

Each face removes half the generation per unit area of the whole slab.

The insulation carries this constant flux, so

$$S_e \delta = h(T_o - T_c) = \frac{k_i}{t}(T_i - T_o).$$

Hence the outer surface and interface temperatures are

$$T_o = T_c + \frac{S_e \delta}{h},$$

$$T_i = T_c + \frac{S_e \delta}{h} + \frac{S_e \delta t}{k_i}.$$

Both expressions return kelvin.

The overall generation fixes the surface heat flux, so k_s affects the temperature gradient inside the slab rather than T_o.

For example the centre temperature is

$$T_i + \frac{S_e \delta^2}{2k_s},$$

also in kelvin.

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Exercise 3. Temperature-dependent conductivity

Data

Quantity	Symbol	Value
Wall thickness	L	$1.2\ \mathrm{m}$
Hot-face temperature	T_h	$925\ \mathrm{K}$
Outside air	T_∞	$300\ \mathrm{K}$
Outside coefficient	h	$23\ \mathrm{W\,m^{-2}\,K^{-1}}$
Conductivity	$k(T)$	$0.073 + 3.942 \times 10^{-4} T\ \mathrm{W\,m^{-1}\,K^{-1}}$
Pipe temperature limit	T_{limit}	$650\ \mathrm{K}$

We need the outward heat flux and the depth at which the wall reaches the pipe-temperature limit.

Assumptions

- Steady one-dimensional conduction with no heat generation.
- Negligible disturbance of the wall temperature field by the embedded pipe.
- The given conductivity relation applies throughout the temperature range.

Calculations

Write $k(T) = a + bT$, where $a = 0.073\ \mathrm{W\,m^{-1}\,K^{-1}}$ and $b = 3.942 \times 10^{-4}\ \mathrm{W\,m^{-1}\,K^{-2}}$.

Let x increase from the hot face towards the air, $L = 1.2\,\mathrm{m}$, and $T_o = T(L)$.

With no generation the positive heat flux is constant:

$$q'' = -k(T)\frac{dT}{dx}.$$

Integrating across the wall and applying convection gives

$$\begin{aligned} q''L &= a(925 - T_o) + \frac{b}{2}(925^2 - T_o^2), \\ q'' &= 23(T_o - 300). \end{aligned} \tag{1}$$

Substituting the second relation into the first produces

$$1.971 \times 10^{-4} T_o^2 + 27.673 T_o - 8516.17 = 0.$$

The physical root is $T_o \approx 307.07\,\mathrm{K}$.

The other root is negative and lies outside the specified temperature interval.

From Eq. 1, the outward heat flux is

$$q'' = 23(T_o - 300) \approx 162.6\,\mathrm{W\,m^{-2}}.$$

Integrating from $925\,\mathrm{K}$ to $650\,\mathrm{K}$ gives

$$q'' x_{650} = a(925 - 650) + \frac{b}{2}(925^2 - 650^2),$$

$$x_{650} \approx 0.648\,\mathrm{m}.$$

Since $k(T) > 0$ throughout the wall, the temperature decreases with x.

The pipe must lie farther than $0.648\,\mathrm{m}$ from the hot face towards the cold face.

Equality gives $650\,\mathrm{K}$ and does not meet the strict limit.

A finite pipe must have its entire occupied region on the cold side if every part must remain below the limit.

Answer

The outward heat flux is $163\,\mathrm{W\,m^{-2}}$. The pipe must be placed more than $0.648\,\mathrm{m}$ from the hot face towards the cold face to remain below $650\,\mathrm{K}$.

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