Workshop 11: Convective mass transfer and dissolution

Attempt these problems before opening the worked solutions.

1 Removing a polymer coating

EXERCISE

Exercise 1. Solvent flowing over a flat coating

In a manufacturing process, an organic solvent (methyl ethyl ketone, MEK) is used to dissolve a thin coating of polymer film away from a nonporous flat surface (length 20 cm, width 10 cm, see Figure 1). The thickness of the polymer film is initially uniform at 0.2 mm. In the process, MEK is added at a flow rate of 30 cm^3/s to an open flat pan of length 30 cm and width 10 cm. The depth of the liquid MEK in the pan is maintained at 2.0 cm. Assuming that the concentration of the dissolved polymer in the bulk solvent is essentially zero ($c_{A\infty} = 0$), determine the rate of removal of polymer coating.

Data: diffusion coefficient of dissolved polymer in MEK = 3 x 10^{-6} cm^2/s; kinematic viscosity of MEK = 6 x 10^{-3} cm^2/s; density of polymer material (solid) = 1.05 g/cm^3; density of MEK = 0.8 g/cm^3; maximum solubility of polymer in MEK = 0.04 g/cm^3.

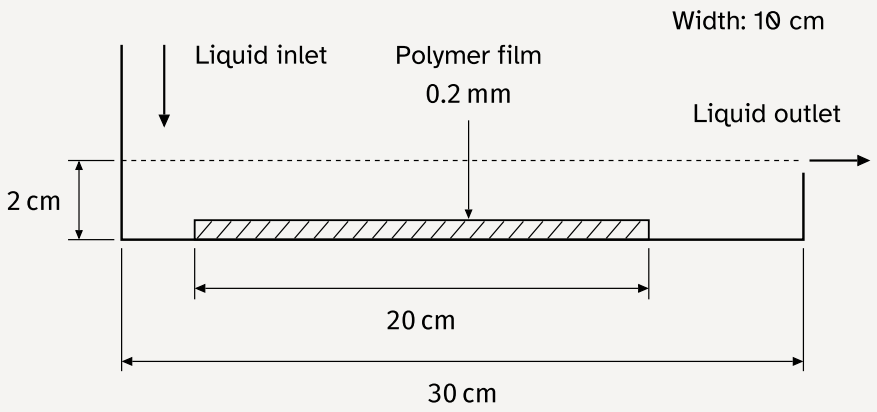


Figure 1: Pan and polymer coating. The pan is 30 cm long and 10 cm wide, the coating is 20 cm long and initially 0.2 mm thick, and the liquid depth is 2 cm. Original redrawing of Figure 1 from the source question sheet.

See solution for Exercise 1

2 Dissolution from a tube wall

EXERCISE

Exercise 2. Balance and a conditional diffusivity estimate

An experiment on the dissolution of fatty deposits in blood vessels involves the setup of a tube (0.8 cm inner diameter, 50 cm in length) that is coated on the inner surface with a thin layer (0.01 cm thick) of lipophilic material that represents the fatty deposits, called species A (ρ = 1.10 $\mathrm{g/cm^3}$). A synthetic solution (ν = 0.02 $\mathrm{cm^2/s}$, ρ = 1.04 $\mathrm{g/cm^3}$) is pumped through the tube at a rate of 20 $\mathrm{cm^3/s}$. The maximum solubility of A in the synthetic solution is 20 $\mathrm{mg\ A/cm^3}$ solution.

a. Develop a material balance to predict the concentration of A in the solution exiting the tube

b. Experimental measurements indicate that the concentration of A in the synthetic solution exiting the tube is 0.1 $\mathrm{mg\ A/cm^3}$. Determine the diffusion coefficient of A in the solution.

See solution for Exercise 2

Solutions

Exercise solutions

Exercise 1. Solvent flowing over a flat coating

Data

Quantity	Symbol	Value
Coated length and width	L, b	$20, 10\ \mathrm{cm}$
Liquid depth and initial coating thickness	H, δ	$2.0, 0.020\ \mathrm{cm}$
Solvent volume flow	Q	$30\ \mathrm{cm^3\ s^{-1}}$
Diffusivity	D_{AB}	$3.0 \times 10^{-6}\ \mathrm{cm^2\ s^{-1}}$
Kinematic viscosity	ν	$0.0060\ \mathrm{cm^2\ s^{-1}}$
Surface and bulk mass concentrations	$C_{A,s}, C_{A,b}$	$0.040, 0\ \mathrm{g\ cm^{-3}}$
Polymer density	ρ_p	$1.05\ \mathrm{g\ cm^{-3}}$

Assumptions

Use mass concentration C_A, so $k_c(C_{A,s} - C_{A,b})$ is a mass flux. In centimetre units, C_A in g cm⁻³ and k_c in cm s⁻¹ give g cm⁻² s⁻¹. Assume isothermal, dilute transport with constant properties, negligible reaction in the liquid and rapid dissolution maintaining saturation at the solid-liquid interface. The coating is thin enough to neglect changes in flow geometry during an instantaneous rate calculation.

Model the exposed upper coating face as a laminar flat plate with a fresh concentration boundary layer at its leading edge. Neglect pan end effects, sidewall transfer and the velocity-profile corrections of a shallow channel. This is the source's external-flow approximation, not a resolved pan flow.

Calculations

Use superficial velocity

$$U = \frac{Q}{b(H - \delta)},$$

coated area $A = bL$ and

$$\mathrm{Re}_L = \frac{UL}{\nu}, \qquad \mathrm{Sc} = \frac{\nu}{D_{AB}}.$$

The average coefficient follows

$$\overline{\mathrm{Sh}}_L = 0.664 \mathrm{Re}_L^{1/2} \mathrm{Sc}^{1/3} = \frac{\overline{k}_c L}{D_{AB}}. \tag{1}$$

Use Eq. 1 for $\mathrm{Re}_L < 5 \times 10^5$ and $\mathrm{Sc} > 0.5$. The initial coating mass is $\rho_p A \delta$.

The polymer removal rate uses the coated area rather than the total pan area.

Convert thickness to $\delta = 0.020\,\mathrm{cm}$.

The initial liquid cross-section is

$$10(2.0 - 0.020) = 19.8\,\mathrm{cm}^2,$$

giving

$$U = 1.51515\,\mathrm{cm\,s^{-1}} \qquad \mathrm{Re}_L = 5050.51 \qquad \mathrm{Sc} = 2000$$

The specified laminar correlation is within its stated group ranges.

Using Eq. 1 gives $\overline{\mathrm{Sh}}_L = 594.537$ and $\overline{k}_c = 8.91805 \times 10^{-5}\,\mathrm{cm\,s^{-1}}$.

The exposed coating area is

$$A = bL = 10(20) = 200\,\mathrm{cm}^2,$$

and the dry-bulk concentration difference is $0.040\,\mathrm{g\,cm^{-3}}$. Substitution gives

$$\dot{m}_A = \overline{k}_c (C_{A,s} - C_{A,b}) A = 7.13444 \times 10^{-4}\,\mathrm{g\,s^{-1}} = 7.13444 \times 10^{-7}\,\mathrm{kg\,s^{-1}}.$$

Initial coating mass is

$$1.05(200)(0.020) = 4.20\,\mathrm{g}.$$

At the assumed constant rate, removal time is

$$\frac{4.20}{\dot{m}_A} = 5886.93\,\mathrm{s} = 1.63526\,\mathrm{h}.$$

Report about $1.64\,\mathrm{h}$.

The 2019 solution reports the same rounded time.

The estimate neglects the small velocity change as the coating shrinks and any change in surface behaviour near complete removal.

The outlet pickup estimated from $\dot{m}_A / Q$ is $2.37815 \times 10^{-5}\,\mathrm{g\,cm^{-3}}$, only 0.0595% of saturation.

This supports the approximation of zero bulk concentration.

Solvent density is unnecessary because kinematic viscosity and mass concentration are given.

Answer

The initial polymer removal rate is $7.13 \times 10^{-4}\,\mathrm{g\,s^{-1}}$. Maintaining that rate gives an estimated coating-removal time of $1.64\,\mathrm{h}$.

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Exercise 2. Balance and a conditional diffusivity estimate

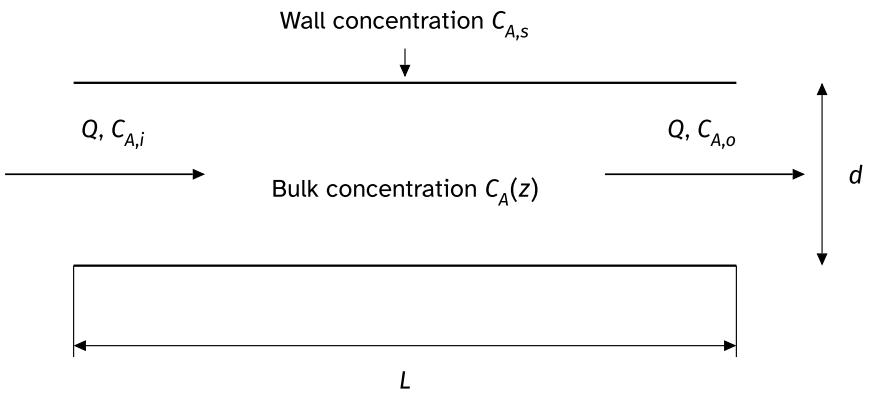


Figure 2: Liquid flows through a tube with a saturated coated wall. The wall adds dissolved material, so the outlet bulk concentration exceeds the inlet concentration.

Data

Quantity	Symbol	Value
Liquid bore diameter	d	$0.80\ \mathrm{cm}$
Coated length	L	$50\ \mathrm{cm}$
Volume flow	Q	$20\ \mathrm{cm^3\ s^{-1}}$
Kinematic viscosity	ν	$0.020\ \mathrm{cm^2\ s^{-1}}$
Saturated wall concentration	$C_{A,s}$	$20\ \mathrm{mg\ cm^{-3}}$
Inlet and measured outlet concentrations	$C_{A,i}, C_{A,o}$	$0, 0.10\ \mathrm{mg\ cm^{-3}}$

The supplied solution uses $\mathrm{Sh}_d = k_c d / D_{AB} = 3.66$ for fully developed laminar mass transfer. The calculation below checks that assumption. It treats the quoted diameter as the initial liquid bore and takes the inlet concentration to be zero.

Assumptions

Assume steady liquid transport during the measurement, negligible axial diffusion, no bulk reaction, constant flow rate and a saturated coated wall. The cross-section balance uses a mixed-mean concentration, not a uniform local radial profile. Assume an upstream uncoated length establishes the laminar velocity profile before the coated section. The source correlation 3.66 additionally requires a developed concentration profile at fixed wall concentration. Reynolds number alone does not establish this condition.

Calculations

$$U = \frac{4Q}{\pi d^2}, \qquad \mathrm{Re}_d = \frac{Ud}{\nu}, \qquad \mathrm{Sc} = \frac{\nu}{D_{AB}}.$$

Estimate entrance lengths using

$$L_h \simeq 0.05 \mathrm{Re}_d d, \qquad L_c \simeq \mathrm{Sc} L_h. \tag{2}$$

The tube measurement determines an effective coefficient before any diffusivity correlation is chosen.

For a slice of length dz, the wall adds mass at rate

$$k_c(C_{A,s} - C_A)\pi d\, dz.$$

The steady balance is

$$Q\frac{dC_A}{dz} = k_c \pi d (C_{A,s} - C_A). \tag{3}$$

Integrate Eq. 3 with $C_A(0) = C_{A,i} = 0$.

Define

$$\Lambda = \frac{k_c \pi d L}{Q},$$

a dimensionless transfer measure.

Then

$$\begin{aligned} \frac{C_{A,s} - C_{A,o}}{C_{A,s} - C_{A,i}} &= \exp(-\Lambda), \\ C_{A,o} &= C_{A,s}[1 - \exp(-\Lambda)]. \end{aligned} \tag{4}$$

If the local coefficient varies, the exponential instead contains

$$\frac{\pi d}{Q}\int_0^L k_c(z)\, dz.$$

Thus the inferred constant coefficient is the length average under this balance model.

Using Eq. 4 with consistent mg cm^{-3} units gives

$$\begin{aligned} k_c &= -\frac{Q}{\pi d L}\ln\left(\frac{20 - 0.10}{20 - 0}\right) \\ &= 7.97771 \times 10^{-4}\,\mathrm{cm\,s^{-1}} \\ &= 7.97771 \times 10^{-6}\,\mathrm{m\,s^{-1}}. \end{aligned}$$

The measured mass pickup is

$$Q(C_{A,o} - C_{A,i}) = 2.0\,\mathrm{mg\,s^{-1}}.$$

Velocity is $U = 39.7887\,\mathrm{cm\,s^{-1}}$ and $\mathrm{Re}_d = 1591.55$, consistent with laminar flow.

Conditional on $\mathrm{Sh}_d = 3.66$,

$$D_{AB} = \frac{k_c d}{3.66} = 1.74376 \times 10^{-4}\,\mathrm{cm^2\,s^{-1}} = 1.74376 \times 10^{-8}\,\mathrm{m^2\,s^{-1}}.$$

This reproduces the coordinator's rounded result, but its validity needs a separate check.

The inferred Schmidt number is 114.695.

Using Eq. 2 gives $L_h = 63.662\,\mathrm{cm}$ and

$$L_c = 7301.68\,\mathrm{cm} = 73.0168\,\mathrm{m},$$

far greater than the coated length of $0.50\,\mathrm{m}$.

An upstream uncoated tube can develop the velocity profile, but cannot develop a solute profile before dissolution starts.

The supplied 3.66 model therefore does not provide a justified physical diffusivity for this entrance-region experiment.

The data establish the effective k_c and mass removal rate under the stated balance.

Inferring a reliable diffusivity additionally needs an appropriate developing-flow correlation or a resolved transport model, with the velocity entrance condition specified.

The conditional numerical result must not be reported as a validated measured diffusivity.

Coating thickness and solid density describe the available inventory; liquid density is unnecessary when kinematic viscosity is supplied.

Answer

The measured outlet concentration gives an effective $k_c = 7.98 \times 10^{-6}\,\mathrm{m\,s^{-1}}$ and a mass pickup of $2.0\,\mathrm{mg\,s^{-1}}$. Assuming $\mathrm{Sh} = 3.66$ gives $D_{AB} = 1.74 \times 10^{-8}\,\mathrm{m^2\,s^{-1}}$, but the concentration-entry check fails. This diffusivity is conditional and is not validated by the experiment.

Back to Exercise 2