CHEN3001-TP
Heat Transport - 2

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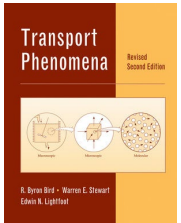
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Outline

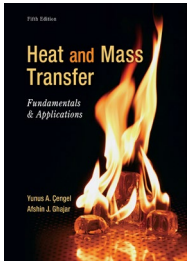
- Conduction - unsteady state heat transfer
  - Lumped capacitance approach (ref-2: book section – 4.1)
    - Biot number
    - Fourier number
    - Application of Biot number
  - Non-lumped capacitance approach (ref-2: book sections – 4.2 and 4.3)
    - Finite system (ref-2: book section – 4.2)
      - Heisler charts (ref-2: book figures – 4.17, 4.18 and 4.19)
    - Semi-infinite system (ref-2: book section – 4.3)

Ref-1: Follow the book "T. P." by Bird, Stewart, and Edwin



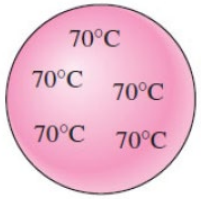
Ref-2: Follow the book "Heat and Mass Transfer" by Cengel and Ghajar

Refer to book section numbers for each topic.

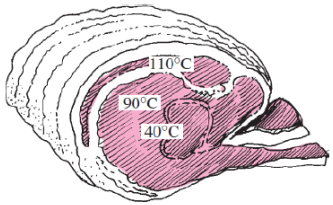


Unsteady conduction

- Conduction in which temperature change with both position and time
- Initiated when a system experiences a change in operating conditions and proceeds until a new steady state (thermal equilibrium) is achieved.
- Two simplifications:
  - Temperature gradients within the solid are neglected - lumped systems – lumped capacitance method

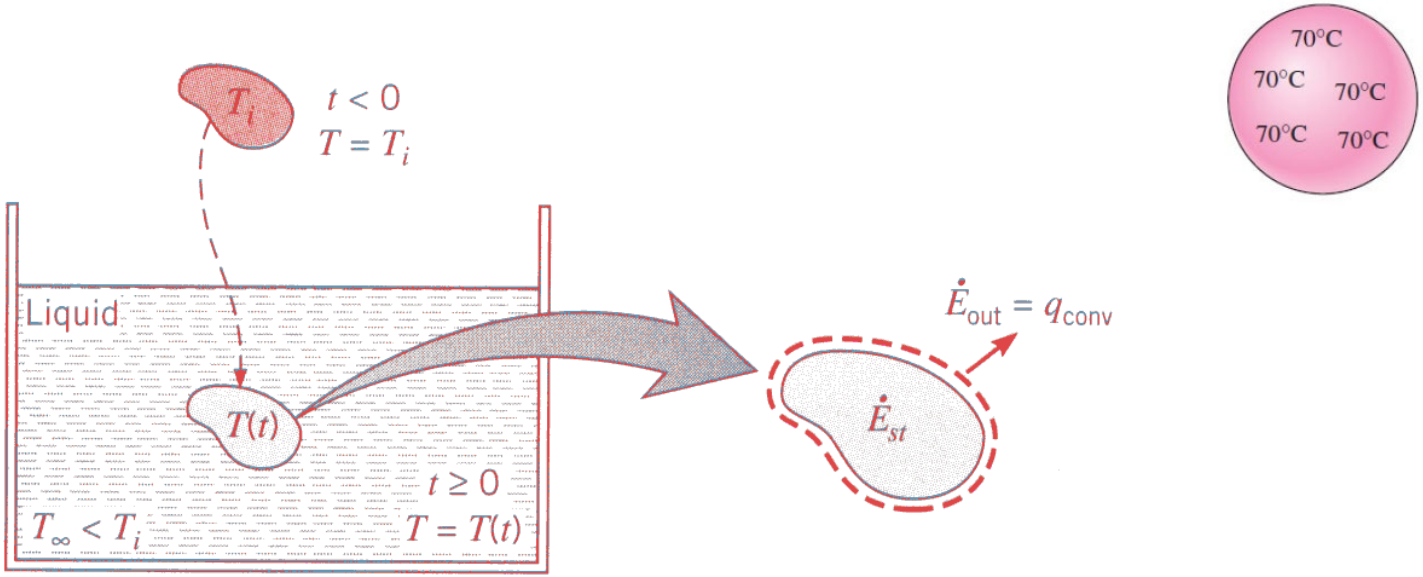


  - 1D heat transfer within the solid - solutions for finite/ semi-infinite solids



Lumped Capacitance Approach

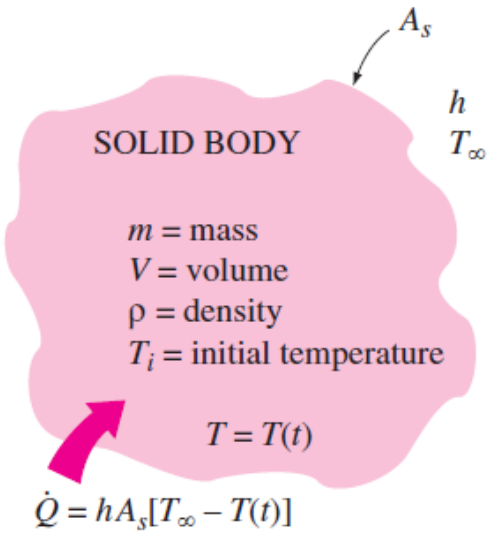
- The simplest approach to unsteady state conduction problems
- Assumptions
  - Resistance in solid is small relative to external resistance
  - Solid temperature is spatially uniform at any instant
- Example – solid immersed in a convective situation



Lumped Capacitance Approach

- Overall energy balance on the solid

$$\left\{\begin{matrix}Heat\ transfer\ into\\ the\ body\ during\ time\\ interval\ dt\end{matrix}\right\} = \left\{\begin{matrix}the\ increase\ in\ the\\ energy\ of\ the\ body\\ during\ dt\end{matrix}\right\}$$



$$-Q_{out} = Q_S$$

$$-hA_s(T - T_\infty) = \rho V C_p \frac{dT}{dt}$$

$$\theta = T - T_\infty \ and \ \theta_i = T_i - T_\infty$$

$$\frac{\rho V C_p}{hA_s}\frac{d\theta}{dt} = -\theta \Rightarrow \frac{\rho V C_p}{hA_s}\int_{\theta_i}^{\theta}\frac{d\theta}{\theta} = -\int_0^t dt$$

$$\boxed{\frac{\rho V C_p}{hA_s} ln\left(\frac{\theta_i}{\theta}\right) = t}$$

Lumped Capacitance Approach

$$\frac{\rho V C_p}{hA_s} ln\left(\frac{\theta_i}{\theta}\right) = t \Longrightarrow \theta = \theta_i \exp\left(-\left(\frac{hA_s}{\rho V C_p}\right) t\right)$$

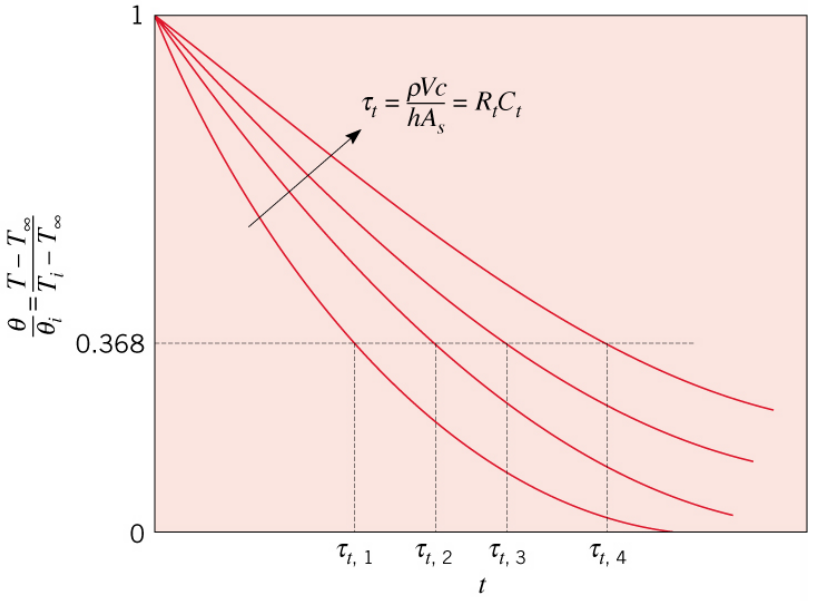
Energy transferred up to time *t:*

$$Q = \int_0^t q dt = hA_s \int_0^t \theta dt$$

$$Q = hA_s \int_0^t \left[\theta_i \exp\left(-\left(\frac{hA_s}{\rho V C_p}\right) t\right)\right] dt$$

$$\tau = \frac{\rho V C_p}{hA_s} = thermal\ time\ constant$$

$$Q = (\rho V C_p)\theta_i \left[1 - exp\left(-\frac{t}{\tau}\right)\right]$$



Lumped Capacitance Approach

Condition of validity – Under what conditions the lumped capacitance approach is valid?

$$Q_{in} - Q_{out} = 0$$

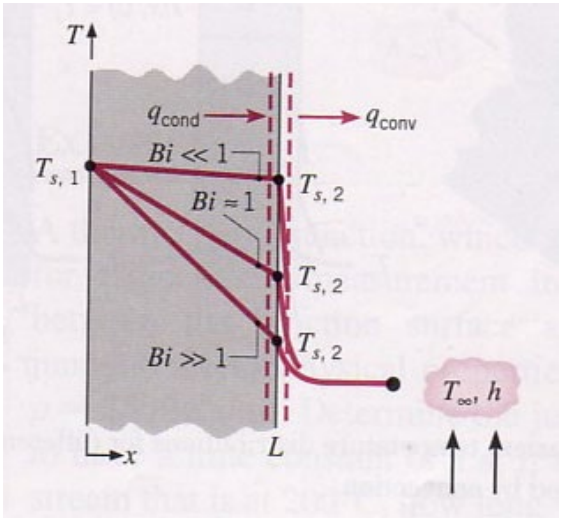
$$\frac{kA}{L}\left(T_{s,1} - T_{s,2}\right) = hA\left(T_{s,2} - T_{\infty}\right)$$

$$\frac{\left(T_{s,1} - T_{s,2}\right)}{\left(T_{s,2} - T_{\infty}\right)} = \frac{L/kA}{1/hA} = \frac{R_{cond}}{R_{conv}} = \frac{hL}{k} = Bi$$

Dimensionless parameter

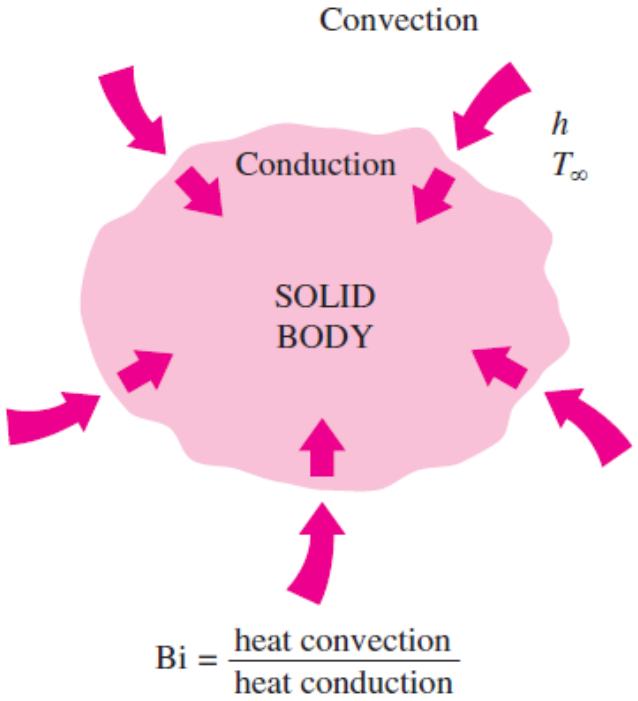
Biot number

S-S conduction, plane wall



Lumped Capacitance Approach

Condition of validity – Under what conditions the lumped capacitance approach is valid?



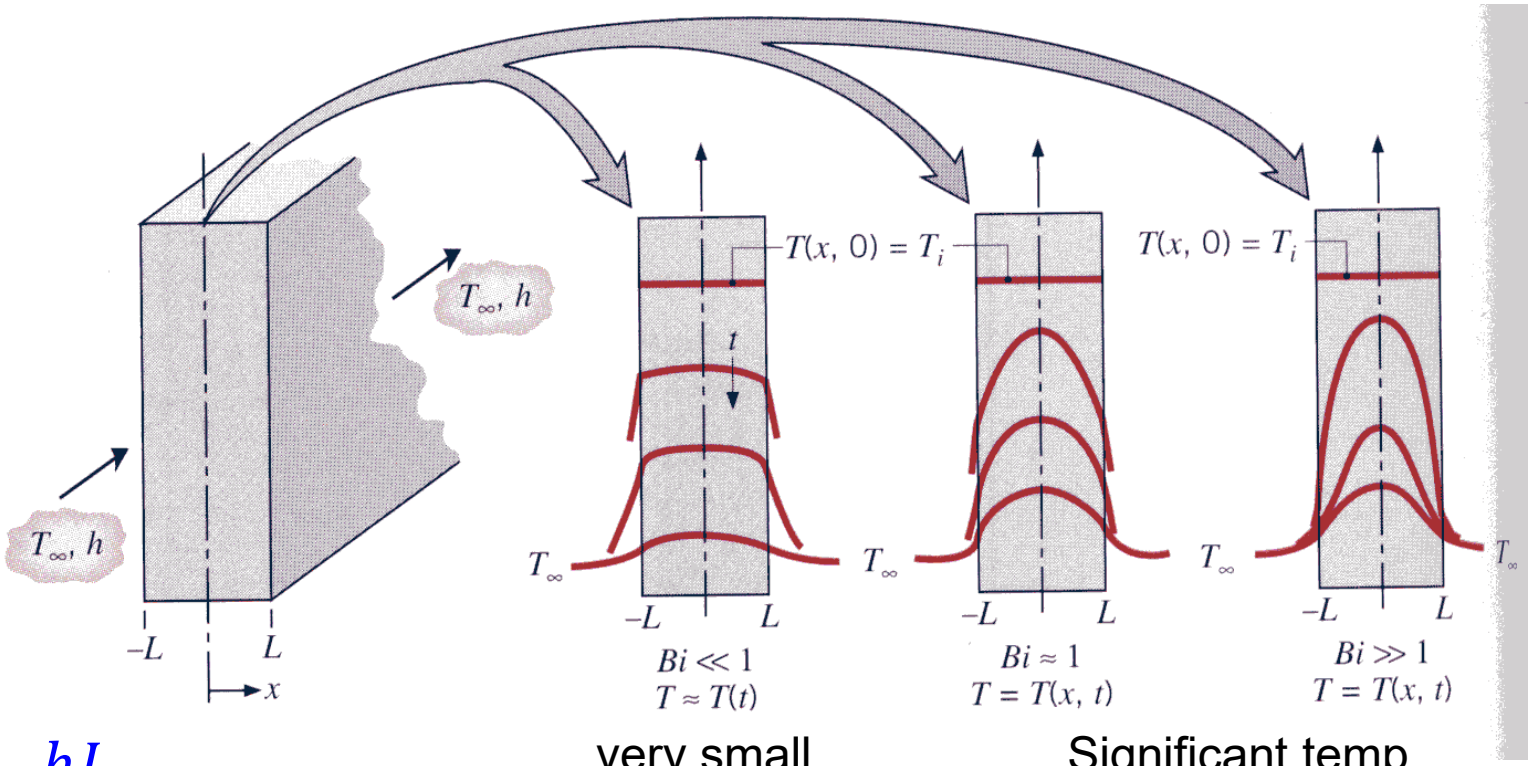
$$\text{Bi} = \frac{h}{k/L_c}\frac{\Delta T}{\Delta T} = \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

$$\text{Bi} = \frac{L_c/k}{1/h} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$

$L_c = \frac{V}{A_s}$ → Characteristic length

Lumped Capacitance Approach

Condition of validity – Under what conditions the lumped capacitance approach is valid?



$$\frac{hL_c}{k} = Bi < 1$$

very small temp gradients

Significant temp gradients

Lumped Capacitance Approach

Biot number and Fourier number

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\left(\frac{hA_s}{\rho V C_p}\right)t\right)$$

$$\frac{hA_s t}{\rho V C_p} = \frac{ht}{\rho C_p L_c} = \frac{hL_c}{k}\,\frac{k}{\rho C_p}\,\frac{t}{{L_c}^2} = Bi\,\frac{\alpha\, t}{{L_c}^2} = Bi \cdot Fo$$

Bi
Biot number

α
Thermal diffusivity

Fo
Fourier number

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp(-Bi \cdot Fo)$$

Lumped Capacitance Approach

Classroom example – 1

A hot spherical particle made out of carbon steel (1.25 cm diameter, 500 °C, k = 40 W/m.K) is cooled by a fan, with convection coefficient of 110 $W/m^2.K$ and air temperature of 25 °C. How long would it take for the sphere to reach a temperature of 100 °C?

Lumped Capacitance Approach

Classroom example – 2

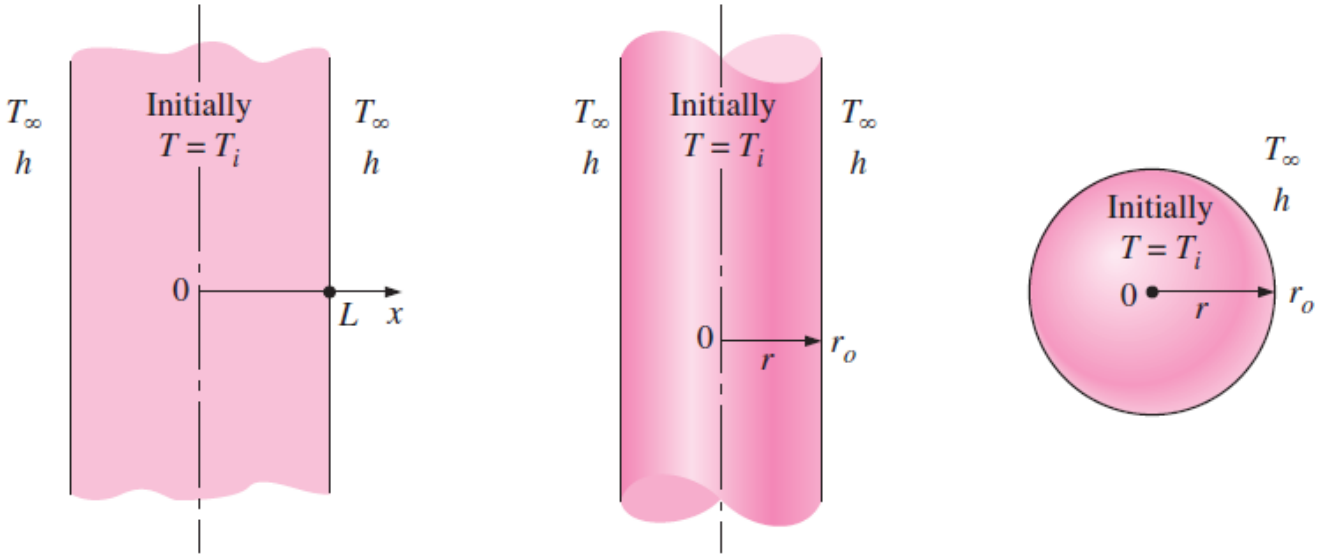
In a manufacturing facility, 5 cm diameter brass spheres (k = 111 W/m.K, r = 8500 kg/m^3, Cp = 385.2 J/kg.K) initially at 121 °C are quenched in a water bath at 20 °C for a period of 2 minutes at a rate of 120 spheres per minute. If the convection heat transfer coefficient 240 W/m^2.K, determine:

- The temperature of the spheres after quenching
- The rate at which heat needs to be removed from the water in order to keep its temperature constant at 20 °C.

Transient Heat Transfer

Non-lumped Capacitance Approach: Finite medium

- What if Bi > 0.1?
- Internal resistance is not negligible
- Consider simple systems → temperature varies with time and position in 1-D problems

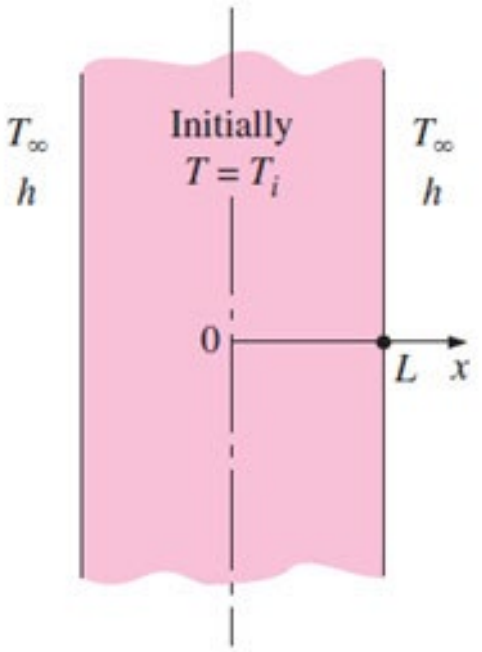


(*a*) A large plane wall (*b*) A long cylinder (*c*) A sphere

Non-lumped Capacitance Approach: Finite medium

Transient conduction through a large plane wall

- Thickness: 2L
- Initially at uniform temperature
- At t = 0, the wall is immersed in a fluid of temperature T_∞
- T(x,t)?
- Assumptions:
  - Constant heat transfer coefficient
  - The height and the width of the wall are large relative to L
  - 1-D approximation is valid
  - There is thermal symmetry at x = 0

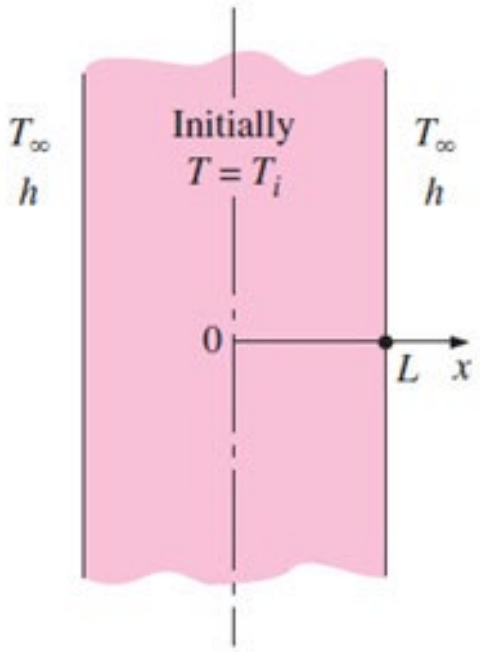


Non-lumped Capacitance Approach: Finite medium

Transient conduction through a large plane wall

Differential equation:

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \Leftrightarrow \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$



Boundary conditions:

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$$

$$-k \left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left(T\big|_{x=L} - T_\infty\right)$$

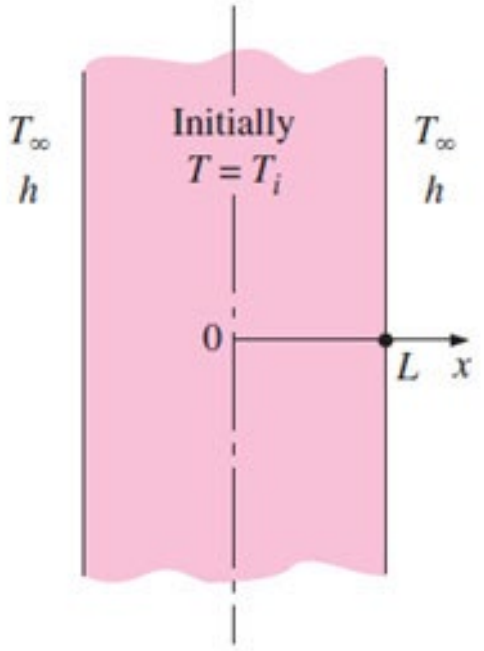
Initial condition: $T|_{t=0} = T_i$ For $0 \leq x \leq L$

Non-lumped Capacitance Approach: Finite medium

Transient conduction through a large plane wall

Non-dimensionalisation of the differential equation:

$$\theta = \frac{T - T_\infty}{T_i - T_\infty}, X = \frac{x}{L}, \tau = \frac{\alpha t}{L^2} = Fourier\ no.$$



$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \Leftrightarrow \frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial X^2}$$

Solution of the above equation:

$$\theta = \sum_{n=1}^{\infty} A_n exp\left(-{\lambda_n}^2 \tau\right)[cos(\lambda_n X)]$$

$$where\ A_n = \frac{4sin\lambda_n}{2\lambda_n + sin(2\lambda_n)} \text{ and } \lambda_n tan\lambda_n = Bi$$

Non-lumped Capacitance Approach: Finite medium

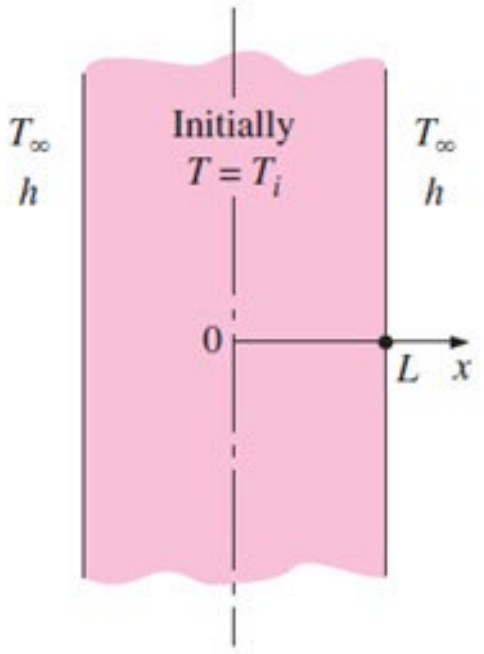
Transient conduction through a large plane wall

Approximate solution:

For Fourier number (τ) > 0.2, the first term of the series is sufficient

$$\theta = \sum_{n=1}^{\infty} A_n exp\left(-{\lambda_n}^2 \tau\right)[cos(\lambda_n X)]$$

$$\theta = A_1 exp\left(-{\lambda_1}^2 \tau\right)[cos(\lambda_1 X)]$$



Non-lump edium

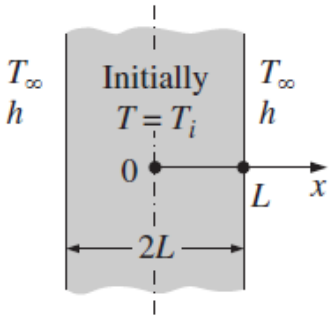
TABLE 4–1

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = hL/k for a plane wall of thickness $2L$, and Bi = hr_o/k for a cylinder or sphere of radius r_o)

	Plane Wall		*Cylinder*		*Sphere*	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

Non-lumped Capacitance Approach: Finite medium

Heisler chart (Plane wall)



$\theta_o = \frac{T_o - T_\infty}{T_i - T_\infty}$

Plate

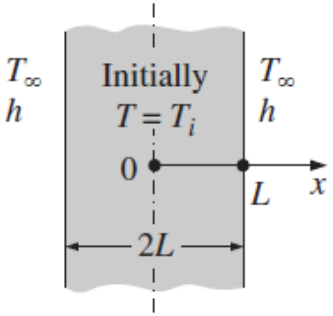
$\frac{k}{hL} = \frac{1}{Bi} =$

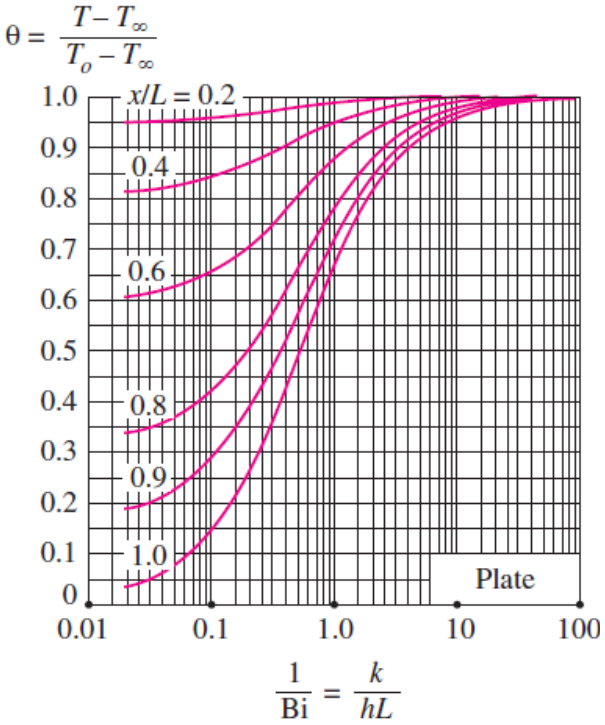
$\tau = \alpha t / L^2$

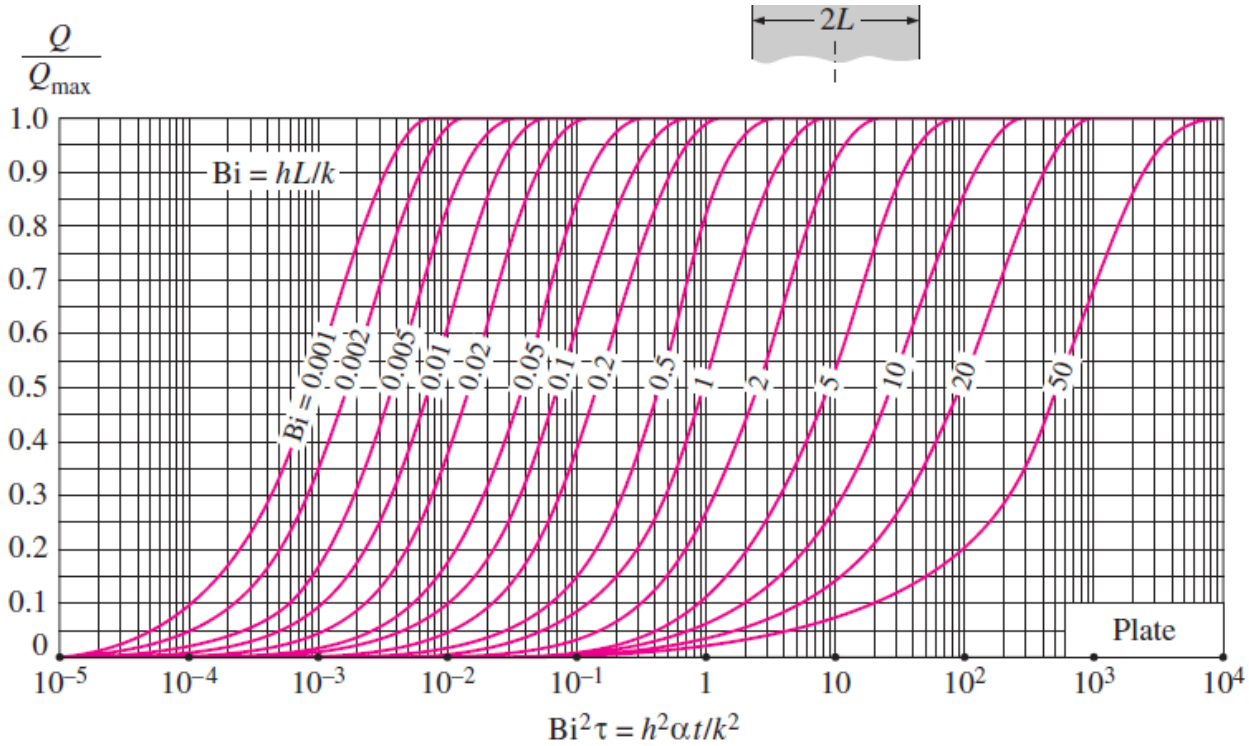
taken from Cengel, 2006

Non-lumped Capacitance Approach: Finite medium

Heisler chart (Plane wall)







taken from Cengel, 2006

Non-lumped Capacitance Approach: Finite medium

Approximate Solutions to 1-D-unsteady state heat conduction model

Plane wall:
$$\theta(x,t)_{\text{wall}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L), \quad \tau > 0.2$$

Cylinder:
$$\theta(r,t)_{\text{cyl}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o), \quad \tau > 0.2$$

Sphere:
$$\theta(r,t)_{\text{sph}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}, \quad \tau > 0.2$$

Center of plane wall ($x = 0$):
$$\theta_{0,\text{wall}} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

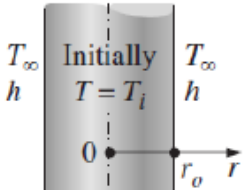
Center of cylinder ($r = 0$):
$$\theta_{0,\text{cyl}} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

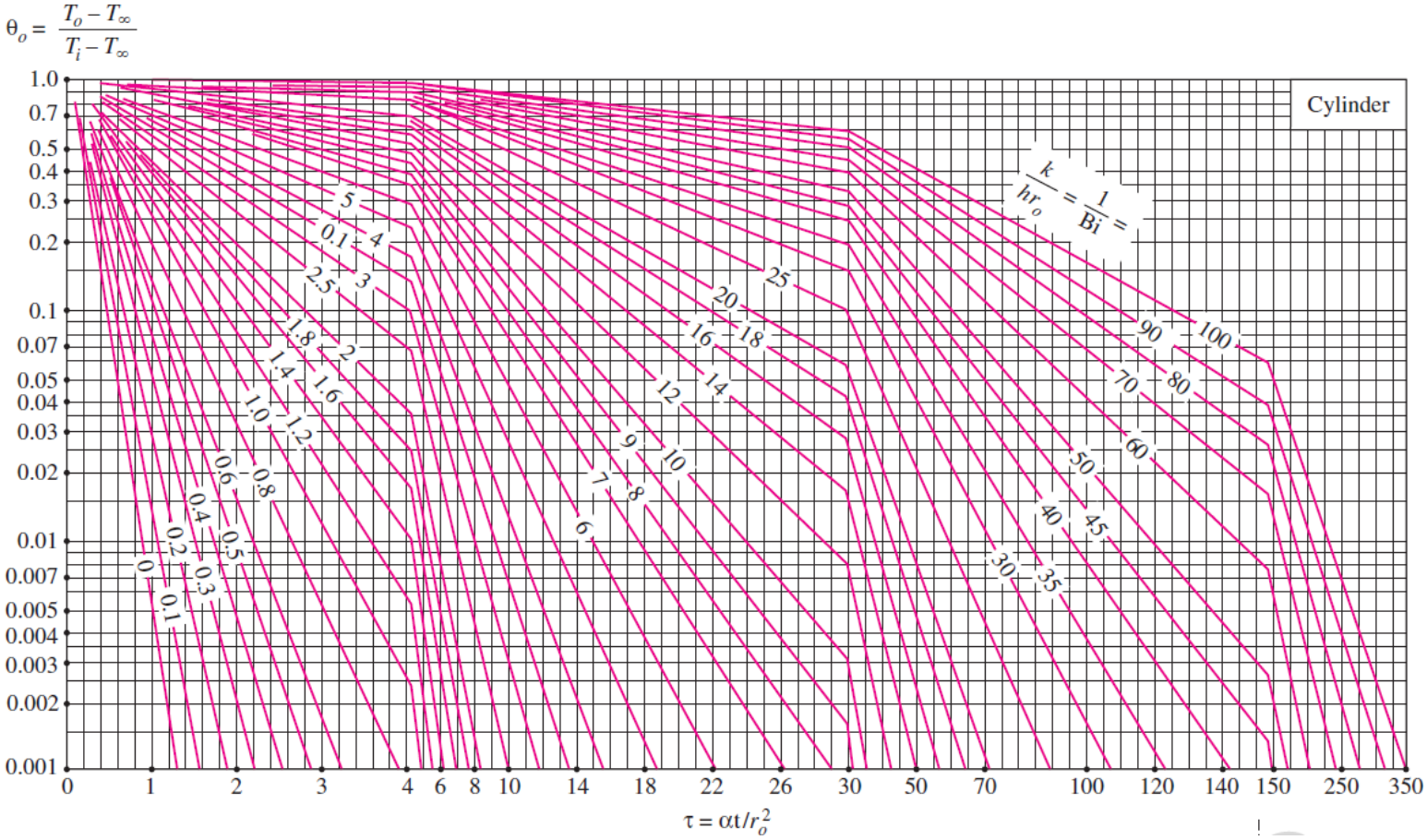
Center of sphere ($r = 0$):
$$\theta_{0,\text{sph}} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

taken from Cengel, 2006

Non-lumped Capacitance Approach: Finite medium

Heisler chart (Cylinder)

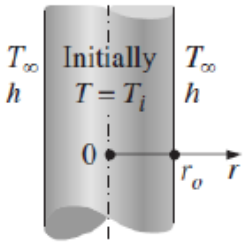


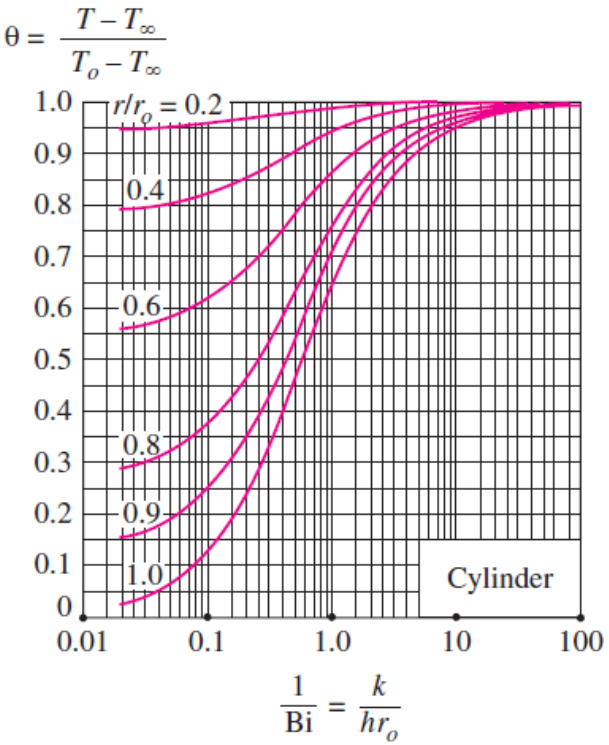


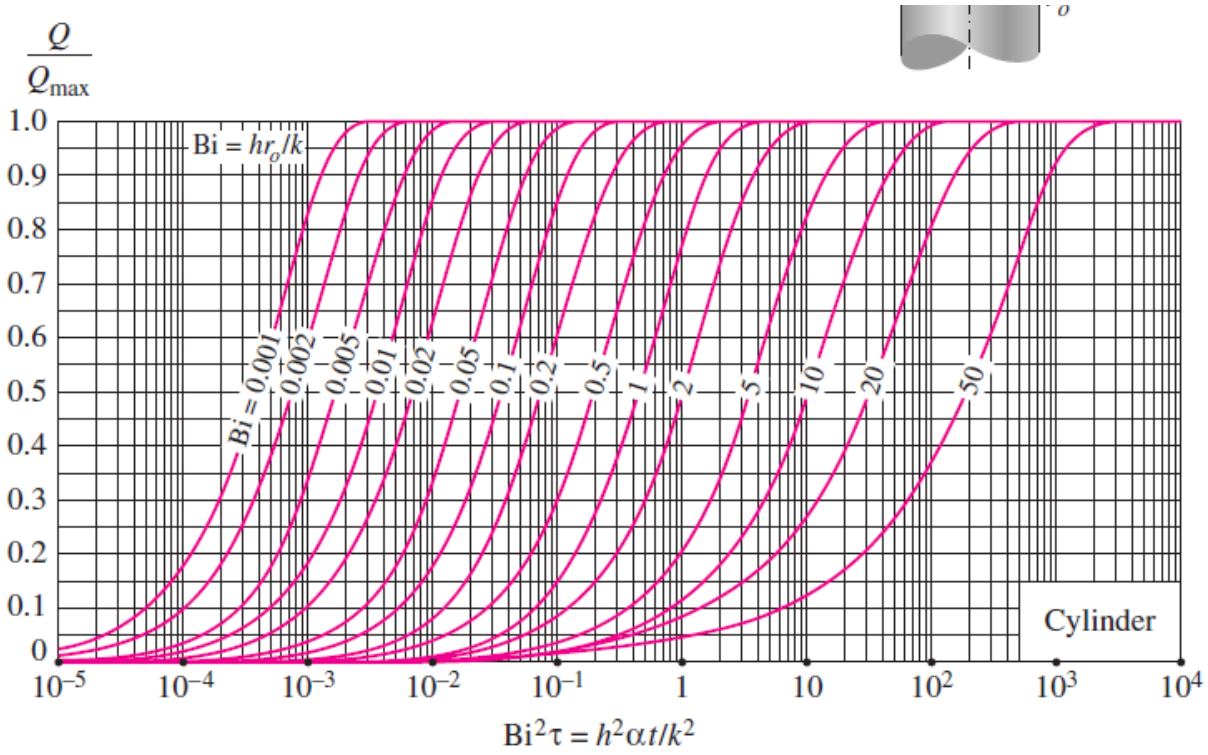
taken from Cengel, 2006

Non-lumped Capacitance Approach: Finite medium

Heisler chart (Cylinder)



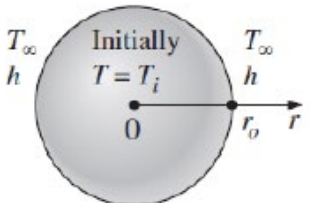


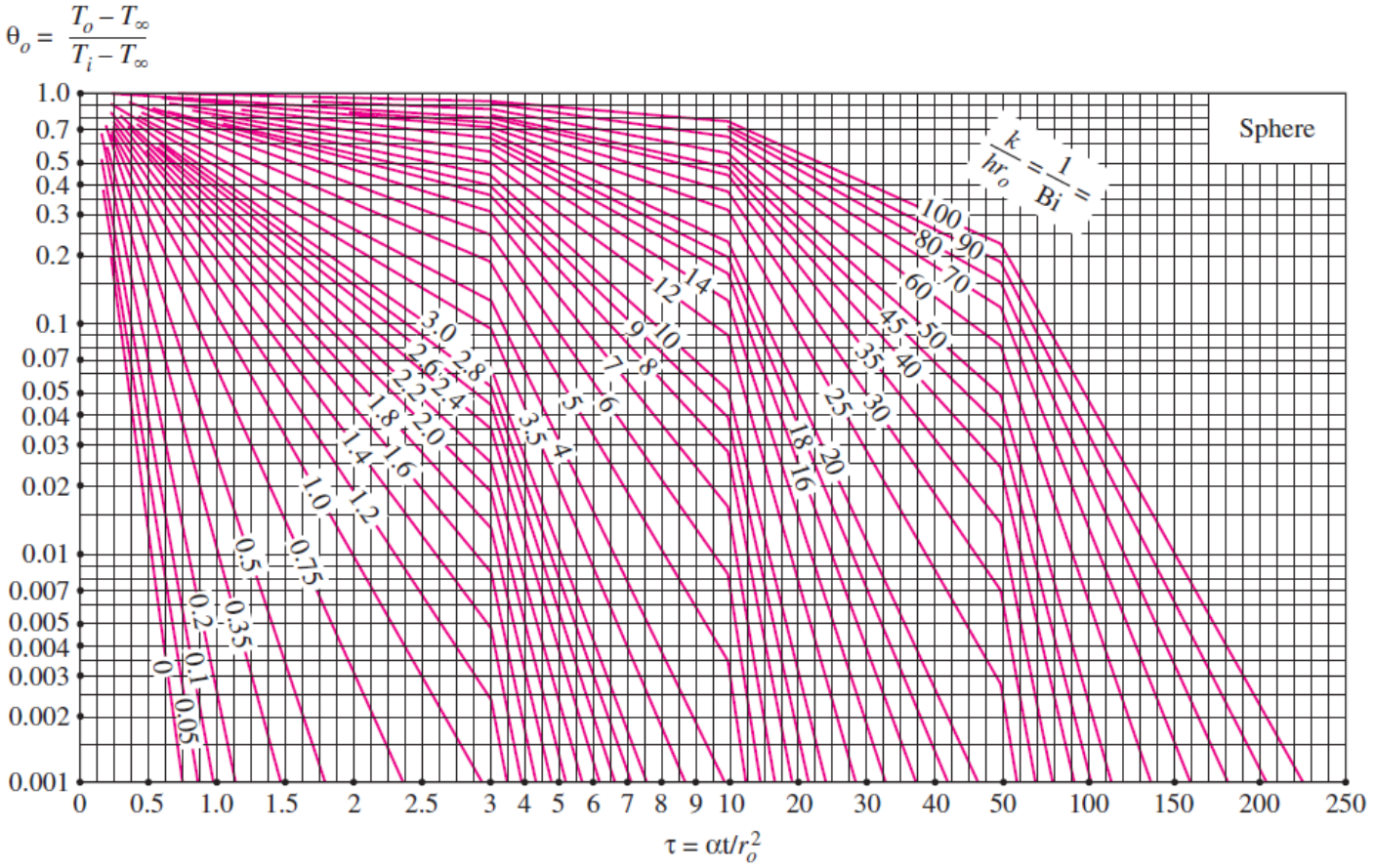


taken from Cengel, 2006

Non-lumped Capacitance Approach: Finite medium

Heisler chart (Sphere)

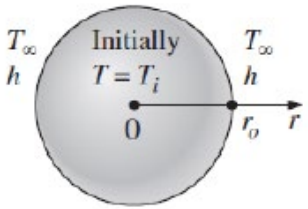


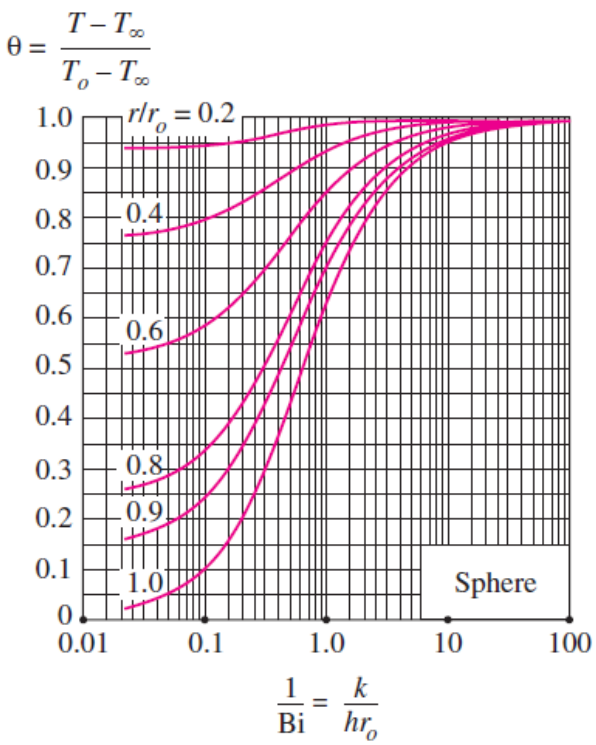


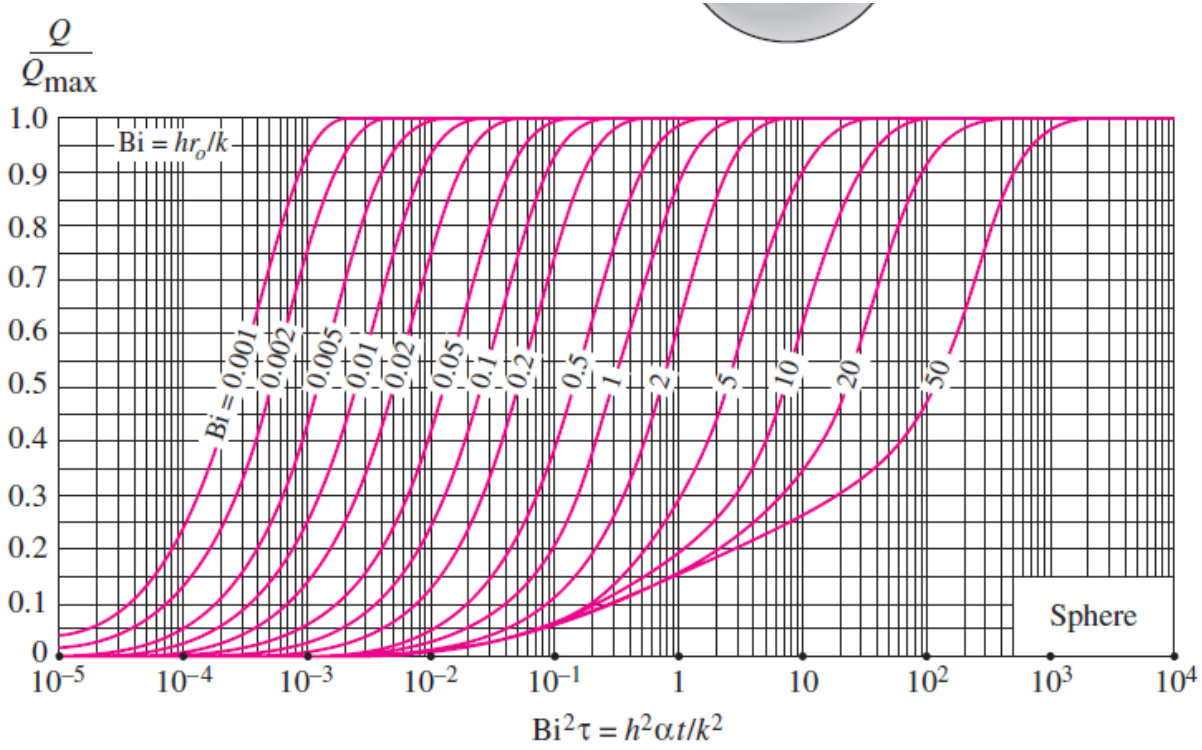
taken from Cengel, 2006

Non-lumped Capacitance Approach: Finite medium

Heisler chart (Sphere)







taken from Cengel, 2006

Non-lumped Capacitance Approach: Finite medium

Classroom example – 3

A flat wall of fire-clay brick (0.5 m thick) initially has a uniform temperature of 200 K. One of its faces is suddenly exposed to hot gas at 1200 K. If the heat transfer coefficient on the hot side is 7.38 $W/m^2.K$ and the other face of the wall is insulated, determine:

- The time it takes for the centre of the wall to reach 600 K
- The temperature of the insulated wall at the time evaluated in (a)

Properties of fire-clay brick:

$k = 1.125$ W/m K

$Cp = 919$ J/kg K

$\rho = 2310$ kg/m^3

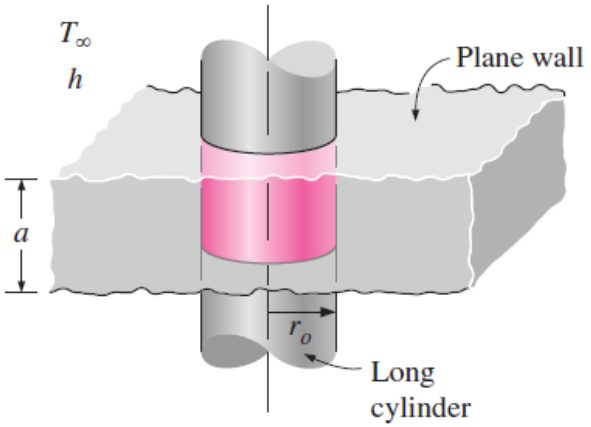
Non-lumped Capacitance Approach: Finite medium

Classroom example – 4

A steel cylinder (D = 30.5 cm, L = 61 cm) is initially at 645 K. It is immersed in an oil bath with a constant temperature of 310K. If the conductance of the steel surface is 34 $W/m^2.K$, what is the temperature::

- at the centre of the cylinder after 1 hour?
- at the centre of the top surface of the cylinder?

Properties of steel:
k = 42.9 W/m K
Cp = 473 J/kg K
ρ = 7820 kg/m^3



Non-lumped Capacitance Approach: Finite medium

Classroom example – 5

An egg, initially at a uniform temperature of 5 °C, is dropped into boiling water at 95 °C. Taking the convection heat transfer coefficient to be 1200 W/m^2.°C, and approximating the egg to be a 5-cm diameter sphere, determine how long it will take for the centre of the egg to reach 70 °C.

Transient Heat Transfer

Non-lumped Capacitance Approach: Semi-infinite medium

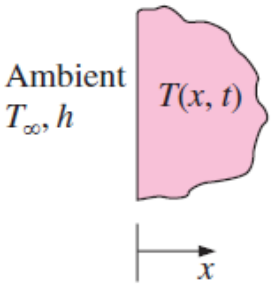
A semi-infinite solid is an idealized body that has a single plane surface and extends to infinity in all directions.

This idealized body is used to indicate that the temperature change in the part of the body in which we are interested (the region close to the surface) is due to the thermal conditions on a single surface.

A thick wall can be modeled as a semi-infinite medium if all we are interested in is the variation of temperature in the region near one of the surfaces, and the other surface is too far to have any impact on the region of interest during the time of observation. The temperature in the core region of the wall remains unchanged in this case.

Transient Heat Transfer

Non-lumped Capacitance Approach: Semi-infinite medium



$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

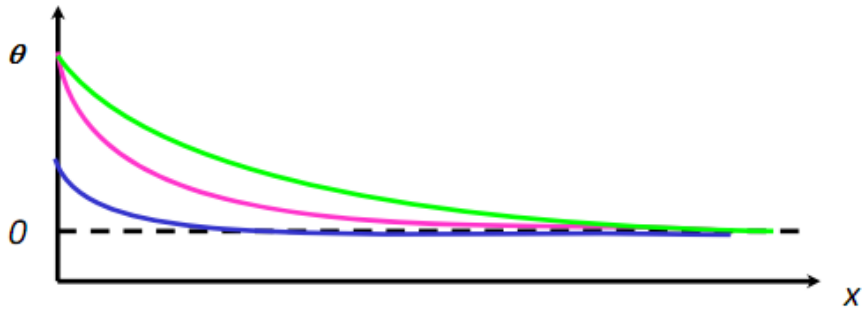
General energy balance equation after neglecting convective and heat source terms + "k" is constant

Boundary conditions:

$At\ x = 0, T(0,t) = T_s\ and\ Q_x = hA(T_s - T_\infty)$

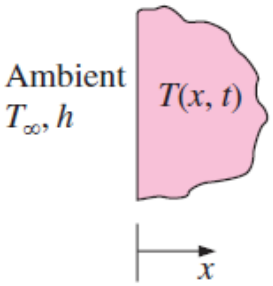
$At\ x = \infty, T(x \to \infty, t) = T_i$

Initial condition: $when\ t = 0, T = T_i$ for all x



Transient Heat Transfer

Non-lumped Capacitance Approach: Semi-infinite medium



$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

General energy balance equation after neglecting convective and heat source terms + "k" is constant

Solution of the above equation (using initial and boundary conditions):

$$\frac{T(x,t) - T_i}{T_\infty - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right)\left[\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$

Where erfc → complementary error function

$$\text{erfc}(\xi) = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\xi} e^{-u^2}\, du$$

If h → ∞ (negligible surface h.t. resistance)

$$\frac{T(x,t) - T_i}{T_\infty - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Transient Heat Transfer

Non-lumped Capacitance Approach: Semi-infinite medium

$$\frac{T(x,t)-T_i}{T_\infty-T_i}=\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)-\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$

If h → ∞ (negligible surface h.t. resistance) ➡ $\frac{T(x,t)-T_i}{T_\infty-T_i}=\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$

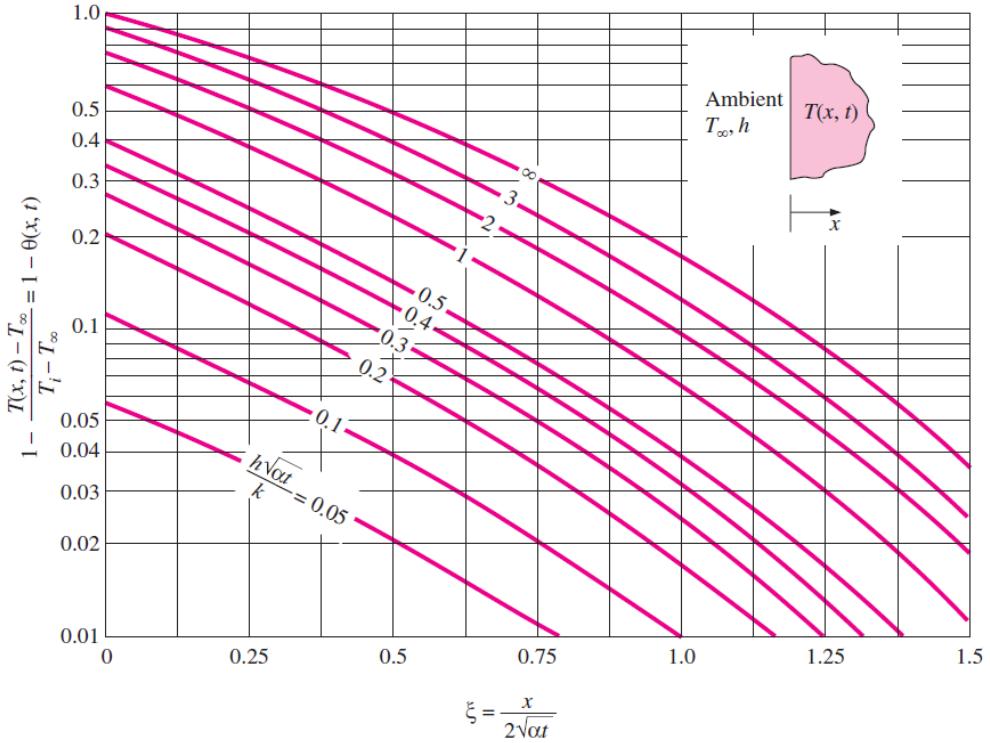


FIGURE 4–23

Variation of temperature with position and time in a semi-infinite solid initially at T_i subjected to convection to an environment at T_∞ with a convection heat transfer coefficient of h (from P. J. Schneider, Ref. 10).

Transient Heat Transfer

Non-lumped Capacitance Approach: Semi-infinite medium

$$\frac{T(x,t)-T_i}{T_\infty-T_i}=\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)-\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$

If h → ∞ (negligible surface h.t. resistance) ➡ $\dfrac{T(x,t)-T_i}{T_\infty-T_i}=\operatorname{erfc}\left(\dfrac{x}{2\sqrt{\alpha t}}\right)$

TABLE 4–3

The complementary error function

ξ	erfc (ξ)	ξ	erfc (ξ)	ξ	erfc (ξ)	ξ	erfc (ξ)	ξ	erfc (ξ)	ξ	erfc (ξ)
0.00	1.00000	0.38	0.5910	0.76	0.2825	1.14	0.1069	1.52	0.03159	1.90	0.00721
0.02	0.9774	0.40	0.5716	0.78	0.2700	1.16	0.10090	1.54	0.02941	1.92	0.00662
0.04	0.9549	0.42	0.5525	0.80	0.2579	1.18	0.09516	1.56	0.02737	1.94	0.00608
0.06	0.9324	0.44	0.5338	0.82	0.2462	1.20	0.08969	1.58	0.02545	1.96	0.00557
0.08	0.9099	0.46	0.5153	0.84	0.2349	1.22	0.08447	1.60	0.02365	1.98	0.00511
0.10	0.8875	0.48	0.4973	0.86	0.2239	1.24	0.07950	1.62	0.02196	2.00	0.00468
0.12	0.8652	0.50	0.4795	0.88	0.2133	1.26	0.07476	1.64	0.02038	2.10	0.00298
0.14	0.8431	0.52	0.4621	0.90	0.2031	1.28	0.07027	1.66	0.01890	2.20	0.00186
0.16	0.8210	0.54	0.4451	0.92	0.1932	1.30	0.06599	1.68	0.01751	2.30	0.00114
0.18	0.7991	0.56	0.4284	0.94	0.1837	1.32	0.06194	1.70	0.01612	2.40	0.00069
0.20	0.7773	0.58	0.4121	0.96	0.1746	1.34	0.05809	1.72	0.01500	2.50	0.00041
0.22	0.7557	0.60	0.3961	0.98	0.1658	1.36	0.05444	1.74	0.01387	2.60	0.00024
0.24	0.7343	0.62	0.3806	1.00	0.1573	1.38	0.05098	1.76	0.01281	2.70	0.00013
0.26	0.7131	0.64	0.3654	1.02	0.1492	1.40	0.04772	1.78	0.01183	2.80	0.00008
0.28	0.6921	0.66	0.3506	1.04	0.1413	1.42	0.04462	1.80	0.01091	2.90	0.00004
0.30	0.6714	0.68	0.3362	1.06	0.1339	1.44	0.04170	1.82	0.01006	3.00	0.00002
0.32	0.6509	0.70	0.3222	1.08	0.1267	1.46	0.03895	1.84	0.00926	3.20	0.00001
0.34	0.6306	0.72	0.3086	1.10	0.1198	1.48	0.03635	1.86	0.00853	3.40	0.00000
0.36	0.6107	0.74	0.2953	1.12	0.1132	1.50	0.03390	1.88	0.00784	3.60	0.00000

Transient Heat Transfer

Non-lumped Capacitance Approach: Semi-infinite medium

Classroom example – 6

What minimum burial depth would you recommend to avoid freezing under conditions for which the soil, initially at a uniform temperature of 15 °C, is subjected to a constant surface temperature of −10 °C for 3 months?

Properties of soil:

$k = 0.52$ W/m.K

$C_p = 1840$ J/kg.K

$\rho = 2050$ kg/m^3