CHEN3001-TP
Heat Transport - 1

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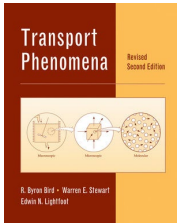
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Outline

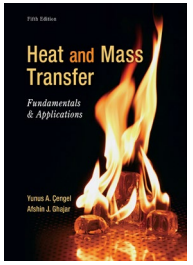
- Introduction
  - Modes of heat transfer (ref-2: book sections – 1.5, 1.6, 1.7 and 1.8)
- Conduction
  - 1-D steady state heat transfer without heat source (ref-2: book sections – 3.1 to 3.4)
  - 1-D steady state heat transfer with heat source
    - Microscopic analysis – Shell heat balance and law of conservation of energy (ref-1: book sections – 10.1 and 10.2)
  - General energy transport equations
    - Application to 1-D steady state example (ref-1: book sections – 11.1)

Ref-1: Follow the book "T. P." by Bird, Stewart, and Edwin

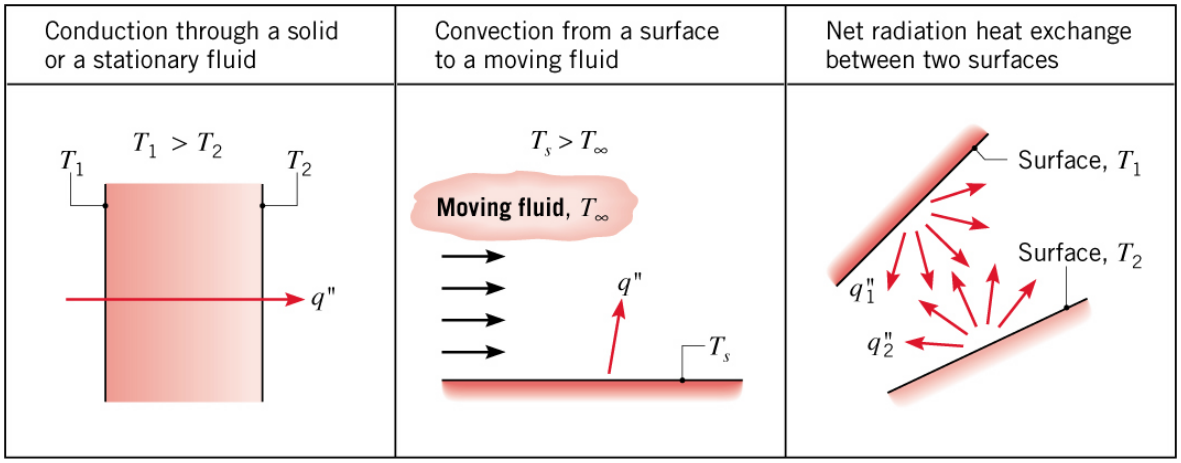


Ref-2: Follow the book "Heat and Mass Transfer" by Cengel and Ghajar

Refer to book section numbers for each topic.



Modes of Heat Transfer



Conduction – Transport of thermal energy in solids and *non-moving fluids* due to short-range atomic interactions, supplemented with the free-electron flow in metals → *Fourier's law*

Convection – Transport of thermal energy between a solid surface (at wall temperature T) and a *moving fluid* (at a far-enough temperature T_∞) → *Newton's law*

Radiation –Transport of thermal energy by far electromagnetic coupling, modelled from the basic black-body theory → *Stefan-Boltzmann law*

Modes of Heat Transfer

Conduction

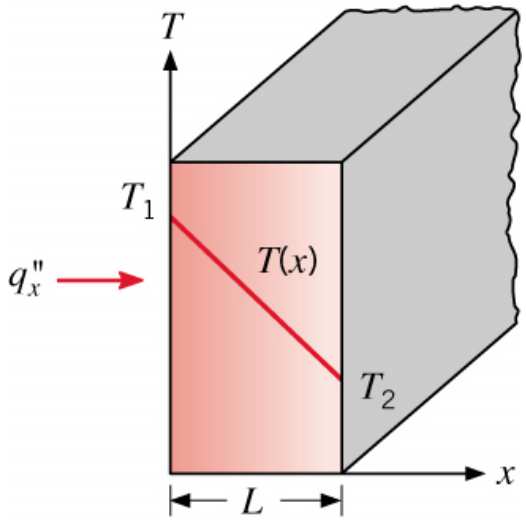
In gas, liquid and solid,

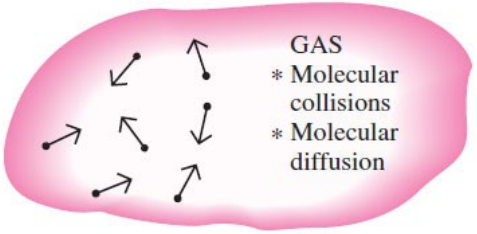
$$q_x = -k\frac{dT}{dx}$$

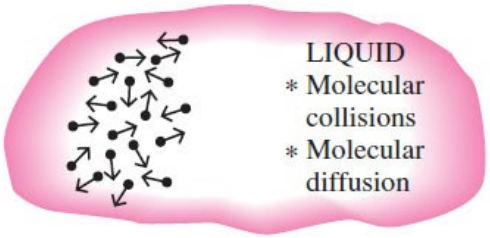
Heat transfer flux

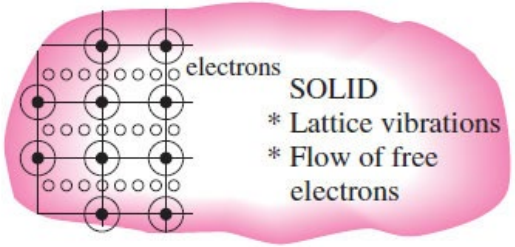
Thermal conductivity

Temperature gradient – driving force









Modes of Heat Transfer

Heat transfer – conduction

$$q_x = -k\frac{dT}{dx}$$

- Thermal conductivity
- Temperature gradient
- Conduction heat flux $\propto$ temperature gradient

Momentum transfer

$$\tau_{yx} = -\mu\frac{dv_x}{dx}$$

- Viscosity
- Velocity gradient
- Viscous flux $\propto$ velocity gradient

Modes of Heat Transfer

Conduction

TABLE 1–1

The thermal conductivities of some materials at room temperature

Material	k, W/m · °C*
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.613
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Glass fiber	0.043
Air (g)	0.026
Urethane, rigid foam	0.026

*Multiply by 0.5778 to convert to Btu/h · ft · °F.

TABLE 1–3

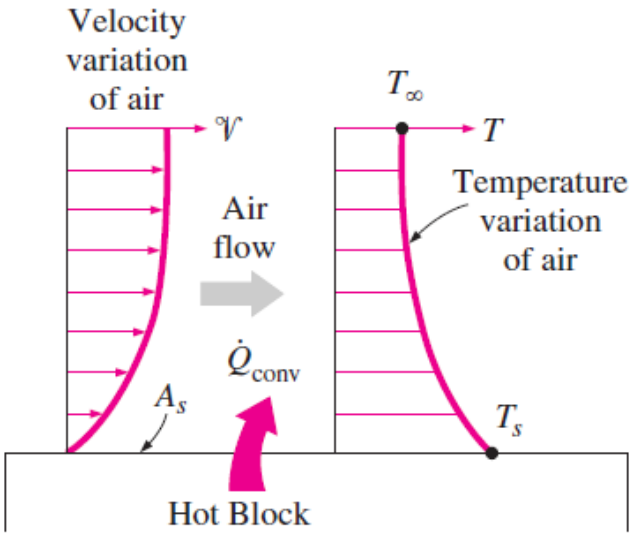
Thermal conductivities of materials vary with temperature

T, K	Copper	Aluminum
100	482	302
200	413	237
300	401	237
400	393	240
600	379	231
800	366	218

Ref: Cengel, Heat & Mass Transfer: A Practical Approach, 2006

Modes of Heat Transfer

Convection



$$Q = hA_s\,(T_s - T_\infty)$$

Convective heat transfer rate

Heat transfer coefficient

Heat transfer surface area

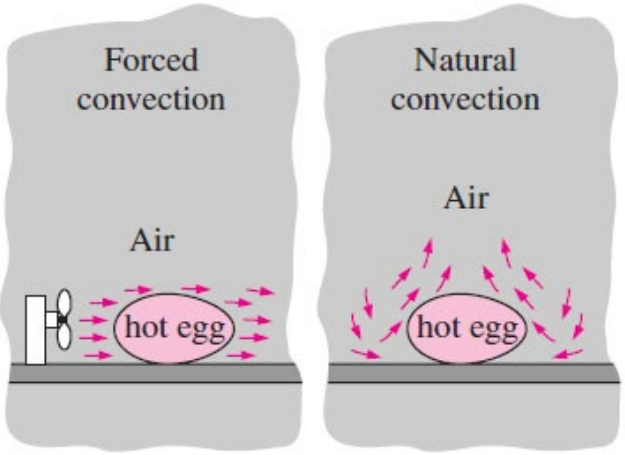


TABLE 1–5

Typical values of convection heat transfer coefficient

Type of convection	h, W/m² · °C*
Free convection of gases	2–25
Free convection of liquids	10–1000
Forced convection of gases	25–250
Forced convection of liquids	50–20,000
Boiling and condensation	2500–100,000

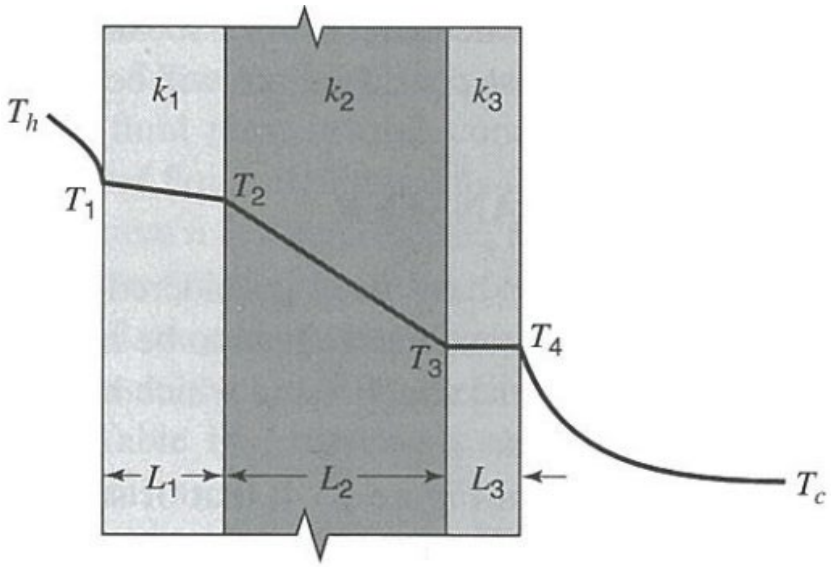
*Multiply by 0.176 to convert to Btu/h · ft² · °F.

1D, Steady State Conduction

Case-1: without heat source

Consider an example of a composite wall:

- Composite wall constructed of three materials in layers.
- Hot gas flows at temperature T_h on 1 side and cool gas flows at T_c on the other side.
- Steady-state heat-transfer rate per unit area?



$$Q_x = h_h A(T_h - T_1)$$ Convective heat transfer at one side

$$= \frac{k_1 A}{L_1}(T_1 - T_2)$$ Conduction in wall-1

$$= \frac{k_2 A}{L_2}(T_2 - T_3)$$

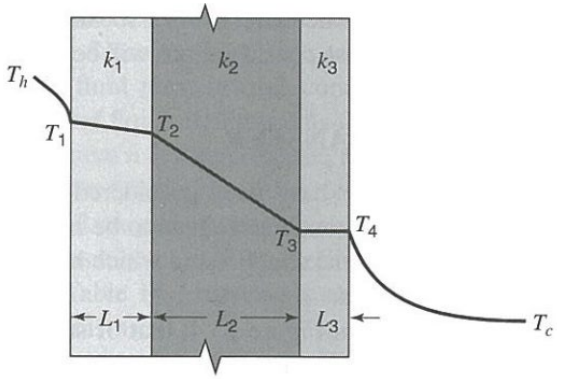
$$= \frac{k_3 A}{L_3}(T_3 - T_4)$$

$$= h_c A(T_4 - T_c)$$ Convective heat transfer at the other side

1D, Steady State Conduction

Case-1: without heat source

Consider an example of a composite wall:



$$Q_x = \frac{(T_h - T_c)}{\left(\frac{1}{h_h A} + \frac{1}{\frac{k_1 A}{L_1}} + \frac{1}{\frac{k_2 A}{L_2}} + \frac{1}{\frac{k_3 A}{L_3}} + \frac{1}{h_c A}\right)}$$

h.t. resistances in series

$$Q_x = \frac{\Delta T}{\sum R} = UA\Delta T \qquad \Rightarrow U = \frac{1}{A\sum R}$$

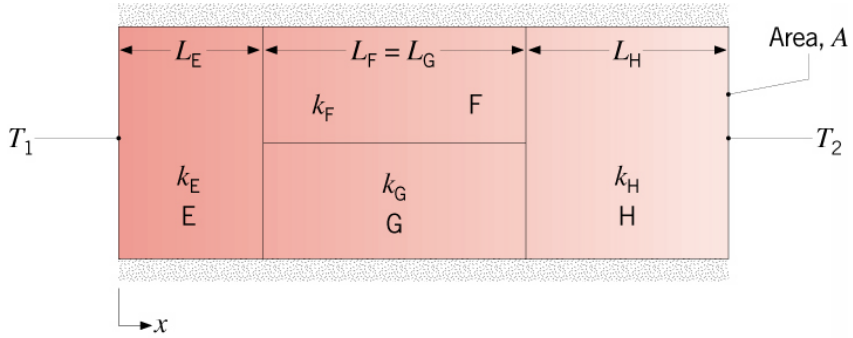
Overall h.t. resistance

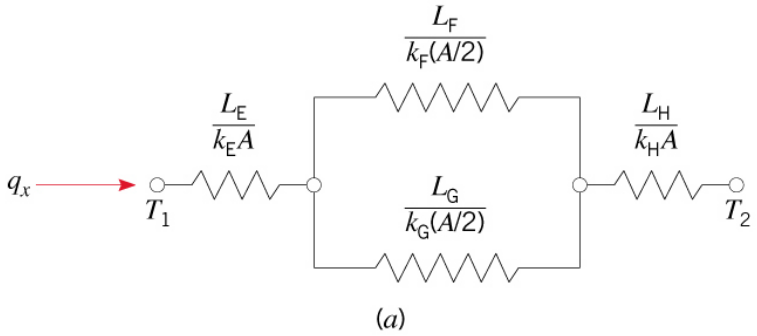
Overall h.t. coefficient

1D, Steady State Conduction

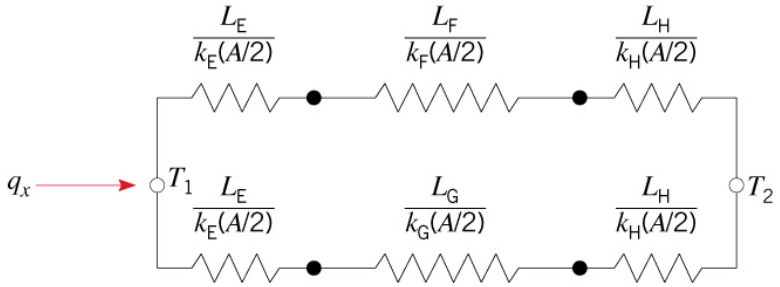
Case-1: without heat source

Consider an example of a series and parallel composite wall:





(a)

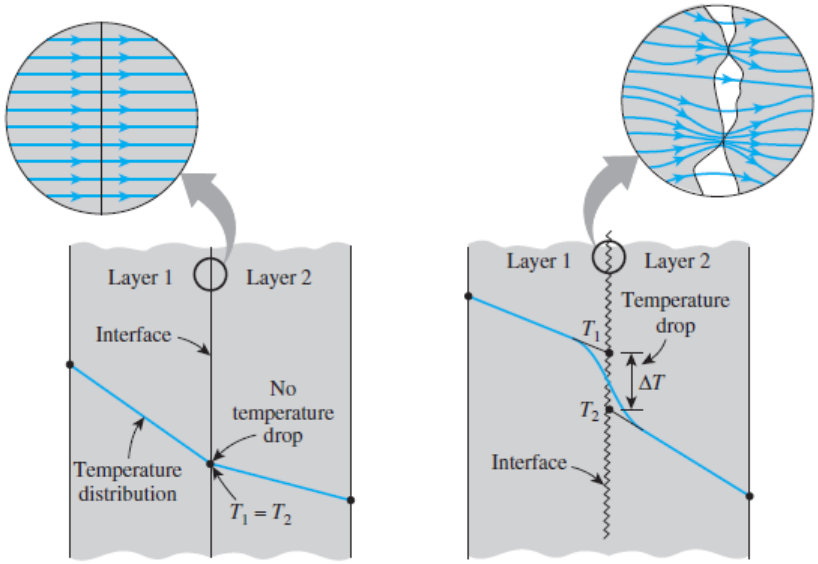


(b)

1D, Steady State Conduction

Case-1: without heat source

Consider an example of thermal contact resistance:



(*a*) Ideal (perfect) thermal contact

(*b*) Actual (imperfect) thermal contact

- Flat surfaces that appear smooth turn out to be rather rough when examined under a microscope with numerous peaks and valleys.
- When two such surfaces are pressed against each other, the peaks form good material contact but the valleys form voids filled with air in most cases.
- An interface contains numerous air gaps of varying sizes that act as insulation because of the low thermal conductivity of air.
- Thus, an interface offers some resistance to heat transfer, and this resistance for a unit interface area is called the thermal contact resistance, Rc.

1D, Steady State Conduction

Case-1: without heat source

Consider an example of thermal contact resistance:

$$Q = Q_{contact} + Q_{gap} = h_c A \Delta T$$

$$R_c = \frac{\Delta T}{Q} = \frac{1}{h_c A} = thermal\ contact\ resistance$$

R_c depends on:

- surface roughness,
- material properties,
- temperature and pressure at the interface, and
- type of fluid trapped at the interface.

1D, Steady State Conduction

Case-1: without heat source

Consider an example of thermal contact resistance:

Thermal contact conductance of some metal surfaces in air (from various sources)

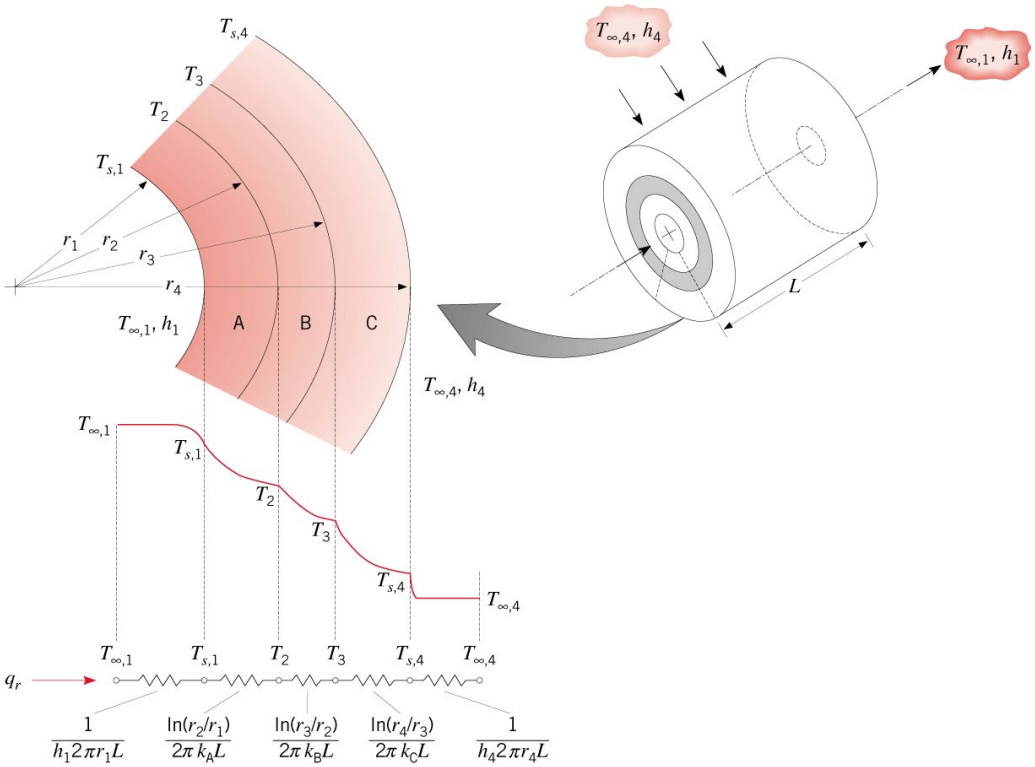
Material	Surface condition	Roughness, μm	Temperature, °C	Pressure, MPa	h_c,* W/m²·K
Identical Metal Pairs					
416 Stainless steel	Ground	2.54	90–200	0.17–2.5	3800
304 Stainless steel	Ground	1.14	20	4–7	1900
Aluminum	Ground	2.54	150	1.2–2.5	11,400
Copper	Ground	1.27	20	1.2–20	143,000
Copper	Milled	3.81	20	1–5	55,500
Copper (vacuum)	Milled	0.25	30	0.17–7	11,400
Dissimilar Metal Pairs					
Stainless steel–Aluminum		20–30	20	10 20	2900 3600
Stainless steel–Aluminum		1.0–2.0	20	10 20	16,400 20,800
Steel Ct-30–Aluminum	Ground	1.4–2.0	20	10 15–35	50,000 59,000
Steel Ct-30–Aluminum	Milled	4.5–7.2	20	10 30	4800 8300
Aluminum-Copper	Ground	1.17–1.4	20	5 15	42,000 56,000
Aluminum-Copper	Milled	4.4–4.5	20	10 20–35	12,000 22,000

1D, Steady State Conduction

Case-1: without heat source

Consider an example of a series of cylinders:

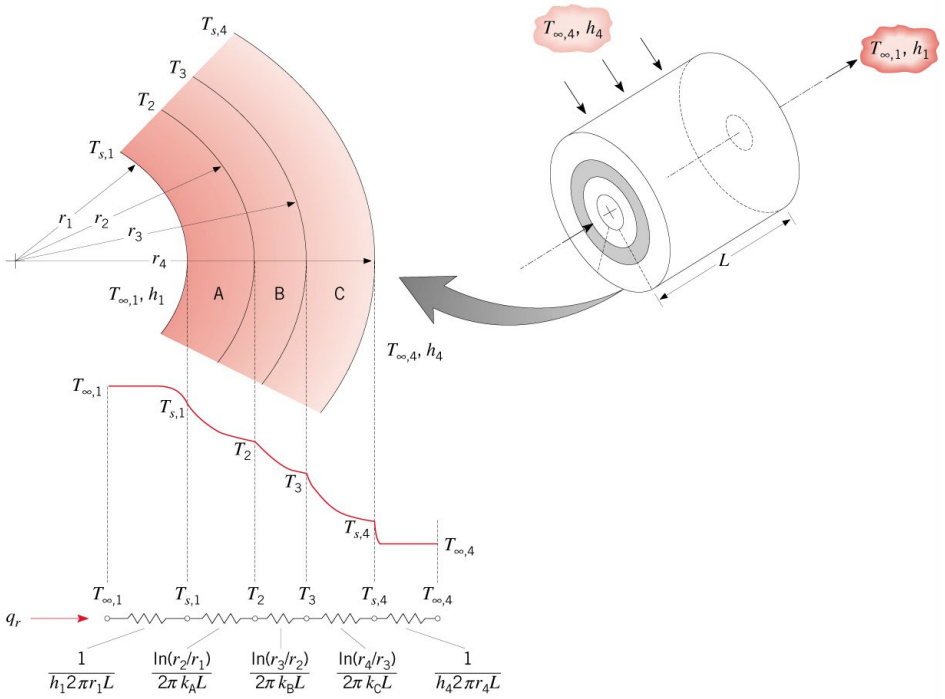
Consider a case of heat conduction across the wall of a hollow cylinder, where hot fluid is passed through the inner passage (r = R1), and cold fluid passes over the outer wall (r = R2). Show that the following temperature distribution is obtained under steady state condition, assuming that the temperatures of the inner and outer walls are constant (Ts,1 and Ts,2, respectively).



1D, Steady State Conduction

Case-1: without heat source

Consider an example of a series of cylinders



$$Q_r = \frac{(T_{\infty,1} - T_{\infty,4})}{\left(\frac{1}{h_1 2\pi r_1 L} + \frac{\ln(r_2/r_1)}{k_A(2\pi L)} + \frac{\ln(r_3/r_2)}{k_B(2\pi L)} + \frac{\ln(r_4/r_3)}{k_C(2\pi L)} + \frac{1}{h_4 2\pi r_4 L}\right)}$$

$$Q_r = -k_A A \frac{dT}{dr}$$

$$\frac{Q_r}{A} dr = \frac{Q_r}{2\pi r L} dr = -k_A dT$$

$$Q_r \int_{r_1}^{r_2} \frac{dr}{r} = -k_A (2\pi L) dT$$

$$Q_r = \frac{k_A(2\pi L)}{\ln(r_2/r_1)} (T_{s,1} - T_2)$$

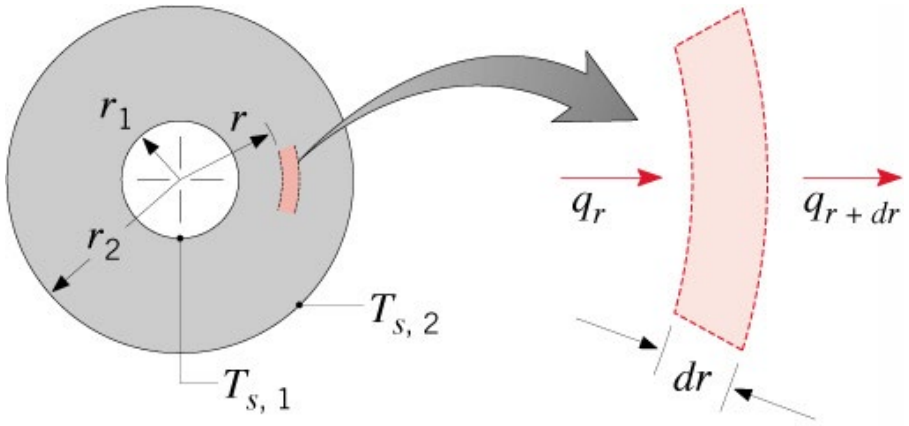
$$Q_r = \frac{(T_{s,1} - T_2)}{R_2}$$

$$R_2 = \frac{\ln(r_2/r_1)}{k_A(2\pi L)}$$

1D, Steady State Conduction

Case-1: without heat source

Consider an example of a series of spheres



$$Q_r = -k_A A \frac{dT}{dr}$$

$$\frac{Q_r}{A} dr = \frac{Q_r}{4\pi r^2} dr = -k_A dT$$

$$Q_r \int_{r_1}^{r_2} \frac{dr}{r^2} = -k_A (4\pi) dT$$

$$Q_r = \frac{k_A (4\pi)}{(1/r_1 - 1/r_2)} (T_{s,1} - T_{s,2})$$

$$Q_r = \frac{(T_{s,1} - T_{s,2})}{R}$$

$$R = \frac{(1/r_1 - 1/r_2)}{4\pi k_A}$$

1D, Steady State Conduction

Case-1: without heat source

	Plane wall	Cylindrical wall	Spherical wall
Heat equation	$k\frac{d^2T}{dx^2}=0$	$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)=0$	$\frac{1}{r^2}\frac{d}{dr}\left(kr^2\frac{dT}{dr}\right)=0$
Heat rate	$\frac{kA}{L}\Delta T$	$\frac{2\pi Lk}{\ln(r_2/r_1)}\Delta T$	$\frac{4\pi k}{(1/r_1)-(1/r_2)}\Delta T$
Heat flux	$\frac{k}{L}\Delta T$	$\frac{k}{r\ln(r_2/r_1)}\Delta T$	$\frac{k}{r^2\left[(1/r_1)-(1/r_2)\right]}\Delta T$
Thermal resistance	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi Lk}$	$\frac{(1/r_1)-(1/r_2)}{4\pi k}$

1D, Steady State Conduction

Classroom example:

Consider a 0.8-m-high and 1.5-m-wide glass window with a thickness of 8 mm and a thermal conductivity of k = 0.78 W/m °C. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 20 °C while the temperature of the outdoors is -10 °C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be h1 = 10 $W/m^2 \cdot °C$ and h2 = 40 $W/m^2 \cdot °C$, which includes the effects of radiation.

1D, Steady State Conduction

Case-2: with heat source
In this discussion, we will use the shell balance approach.

Shell balance approach:

- Sketch system, choose coordinate system, select a control volume (or a thin "shell") → in the direction of the flux
- Determine all of the relevant energy fluxes
- Divide by the volume of the CV/shell
- Take the limit as the thickness of the shell approaches zero get a differential equation
- Substitute in Fourier's law of conduction, e.g. $q_x = -k\frac{dT}{dx}$
- Solve the equations, by considering boundary conditions, conditions of flow, etc.

1D, Steady State Conduction

Case-2: with heat source

Shell balance approach:

Heat balance over a control volume (CV) or shell

$$\left\{\begin{matrix}Rate\ of\\ change\ of\\ energy\\ accumulatin\end{matrix}\right\} = \left\{\begin{matrix}net\ (in\ -out)\\ convection\\ heat\ flux\end{matrix}\right\} + \left\{\begin{matrix}net\ (in\ -out)\\ conduction\\ heat\ flux\end{matrix}\right\} + \left\{\begin{matrix}rate\ of\\ energy\\ production\end{matrix}\right\}$$

Heat energy that passes through CV boundaries

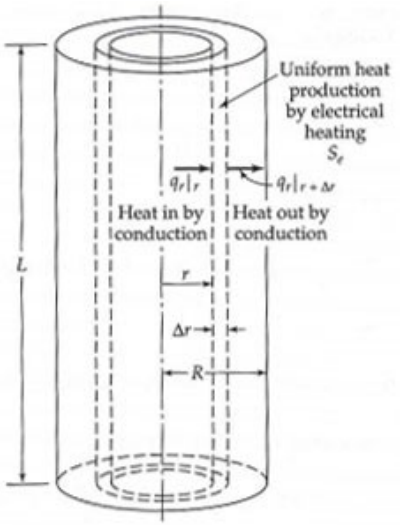
Chemical reaction
Electrical current/heat source
Viscous heat source

1D, Steady State Conduction

Case-2: with heat source

An electric wire with electrical conductivity k_e ohm^{-1}.cm^{-1}. Electric current is passed with current density I amp/cm^2. Some electrical energy is converted into heat (thermal energy). The rate of heat production per unit volume is:

$$S_e = I^2/k_e$$



The surface of the wire is maintained at temperature T_0. Radial temperature distribution?

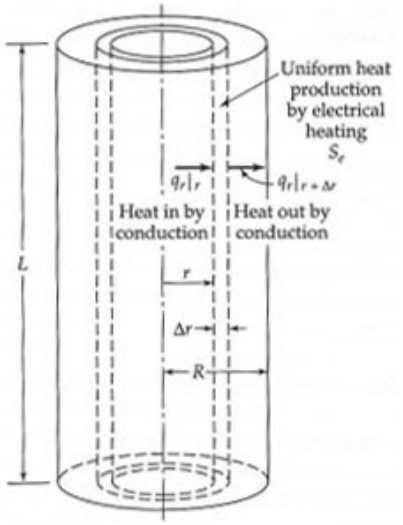
Assume that the thermal and electrical conductivities do not vary greatly in the system.

1D, Steady State Conduction

Case-2: with heat source

Energy balance over a thin shell

- Rate of heat in across the cylindrical surface at $r = 2\pi r L q_r|_r$
- Rate of heat out across the cylindrical surface at $r+\Delta r = 2\pi(r+\Delta r)L q_r|_{r+\Delta r}$
- Rate of thermal energy production = $2\pi r \Delta r L S_e$



$$2\pi r L q_r\Big|_r - 2\pi(r+\Delta r)L q_r\Big|_{r+\Delta r} = 2\pi r \Delta r L S_e$$

$$\frac{(r+\Delta r)q_r|_{r+\Delta r} - r q_r|_r}{\Delta r} = r S_e$$

$$\lim_{\Delta r \to 0} \frac{(r+\Delta r)q_r|_r - r q_r|_{r+\Delta r}}{\Delta r} = r S_e$$

$$\frac{d}{dr}(r q_r) = r S_e$$

1D, Steady State Conduction

Case-2: with heat source

$$\frac{d}{dr}(rq_r) = rS_e$$

Let us integrate, $rq_r = \frac{S_e r^2}{2} + C_1 \Rightarrow q_r = \frac{S_e r}{2} + \frac{C_1}{r}$

Let us apply B.C. that r = 0 then q_r is not infinite, so C_1 must of zero. $\Rightarrow q_r = \frac{S_e r}{2}$

Apply Fourier's law: $q_r = -k\frac{dT}{dr} = \frac{S_e r}{2}$

Integrate the above equation: $T = -\frac{S_e}{4k}r^2 + C_2$

Apply second B.C. that r = R then $T = T_0 \Rightarrow C_2 = T_0 + \frac{S_e}{4k}R^2$

$$T - T_0 = \frac{S_e}{4k}(R^2 - r^2)$$

1D, Steady State Conduction

Case-2: with heat source

$$T - T_0 = \frac{S_e}{4k}(R^2 - r^2)$$

Parabolic temperature distribution in the radial direction
Maximum temperature rise → at r = 0
Average temperature rise:

$$\langle T \rangle = \frac{\int_0^{2\pi} \int_0^R \frac{S_e}{4k}(R^2 - r^2) r dr d\theta}{\int_0^{2\pi} \int_0^R r dr d\theta} + T_0$$

Heat out flow at the surface:

$$Q|_R = q|_R A = q|_R (2\pi RL)$$

General Energy Transport Equations

For a Newtonian fluid, laminar flow condition and constant thermal conductivity (k),

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + u\nabla . T \right) = k\nabla^2 T + S$$

Heat source

Conductive heat transfer

$$k\nabla^2 T = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

Rate of change of temperature in a C.V.

Convective heat transfer

$$u\nabla . T = u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z}$$

General Energy Transport Equations

In Cartesian coordinates,

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+u_x\frac{\partial T}{\partial x}+u_y\frac{\partial T}{\partial y}+u_z\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

In cylindrical coordinates,

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+u_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

$$\left\{\begin{array}{c}\text{rate of}\\ \text{energy}\\ \text{accumulation}\end{array}\right\}+\left\{\begin{array}{c}\text{net}\\ \text{convection}\\ \text{flux}\end{array}\right\}=\left\{\begin{array}{c}\text{net}\\ \text{conduction}\\ \text{flux}\end{array}\right\}+\left\{\begin{array}{c}\text{rate of}\\ \text{energy}\\ \text{production}\end{array}\right\}$$

General Energy Transport Equations

$$\frac{\partial T}{\partial t} + u\nabla.T = \frac{k}{\rho \hat{C}_p}\nabla^2 T + S$$

Thermal diffusivity

A high conductivity material tends to have a more even temperature distribution than one with a low thermal conductivity

A material with low specific heat requires less thermal energy to heat therefore, it heats faster than one with a high specific heat

General Energy Transport Equations

$$\text{Thermal diffusivity} = \frac{k}{\rho \hat{C}_p}$$

The thermal diffusivities of some materials at room temperature

Material	α, m^2/s*
Silver	149×10^{-6}
Gold	127×10^{-6}
Copper	113×10^{-6}
Aluminum	97.5×10^{-6}
Iron	22.8×10^{-6}
Mercury (l)	4.7×10^{-6}
Marble	1.2×10^{-6}
Ice	1.2×10^{-6}
Concrete	0.75×10^{-6}
Brick	0.52×10^{-6}
Heavy soil (dry)	0.52×10^{-6}
Glass	0.34×10^{-6}
Glass wool	0.23×10^{-6}
Water (l)	0.14×10^{-6}
Beef	0.14×10^{-6}
Wood (oak)	0.13×10^{-6}

Cengel, Heat & Mass Transfer: A Practical Approach, 2006

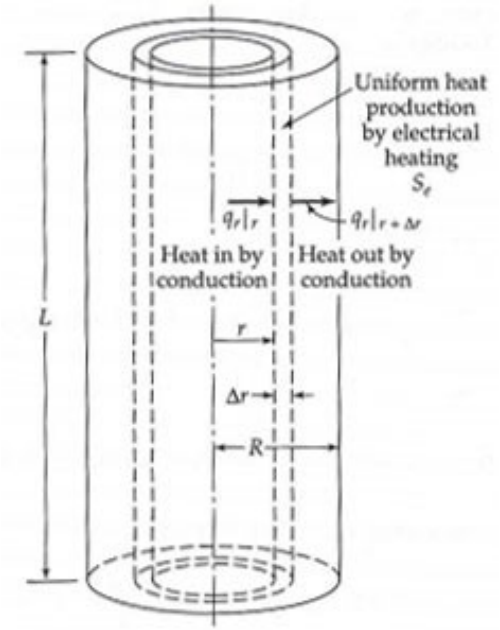
General Energy Transport Equations

1-D steady state problem

Case-2: with heat source

Apply general transport equation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r} \frac{\partial T}{\partial \theta} + u_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$



$$-k \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = S$$

Integrate the above equation:

$$\frac{\partial T}{\partial r} = -\frac{S}{2k} + \frac{C_1}{r}$$

And so on…….