Heat Transfer

Lecture 5: Natural/Free convection

Transport Phenomena

ELECTRONIC WARNING NOTICE FOR COPYRIGHT STATUTORY LICENCES

WARNING

This material has been reproduced and communicated to you by or on behalf of Curtin University in accordance with section 113P of the Copyright Act 1968 (the Act)

The material in this communication may be subject to copyright under the Act. Any further reproduction or communication of this material by you may be the subject of copyright protection under the Act.

Do not remove this notice.

Recap

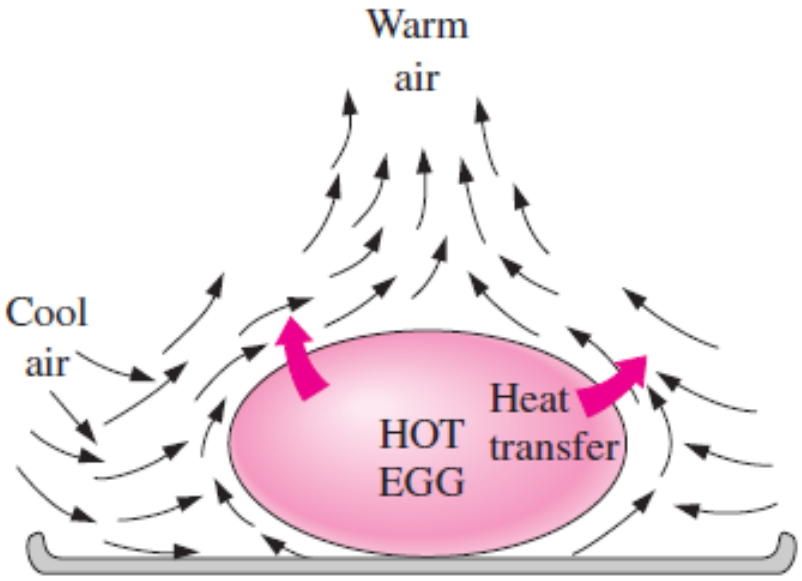
- General energy equations
  - 1D conduction (steady state) without heat source + conjugate heat transfer
  - 1D conduction (steady state) with heat source
    - Analysis using shell balance and using differential equations
  - 1D unsteady state conduction
    - Lumped capacitance
    - Finite medium
    - Semi-infinite medium
- Forced convection
  - Flow over surfaces
    - Prediction based on boundary layer theory
  - Flow in pipes and conduits
    - Constant surface heat flux
    - Constant temperature

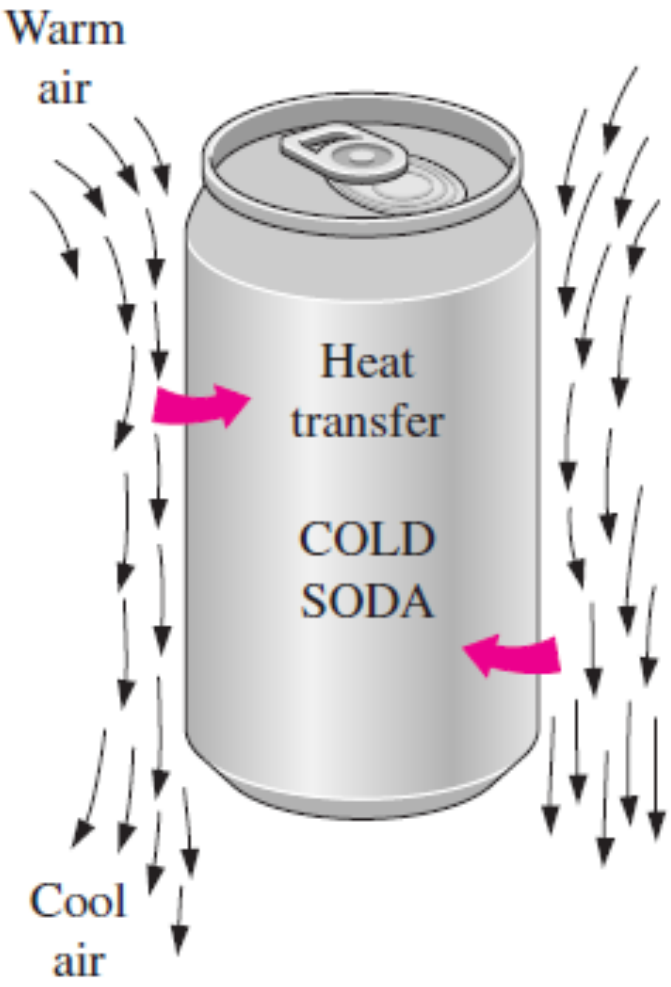
$Nu = f(Re, Pr)$

Learning Outcomes

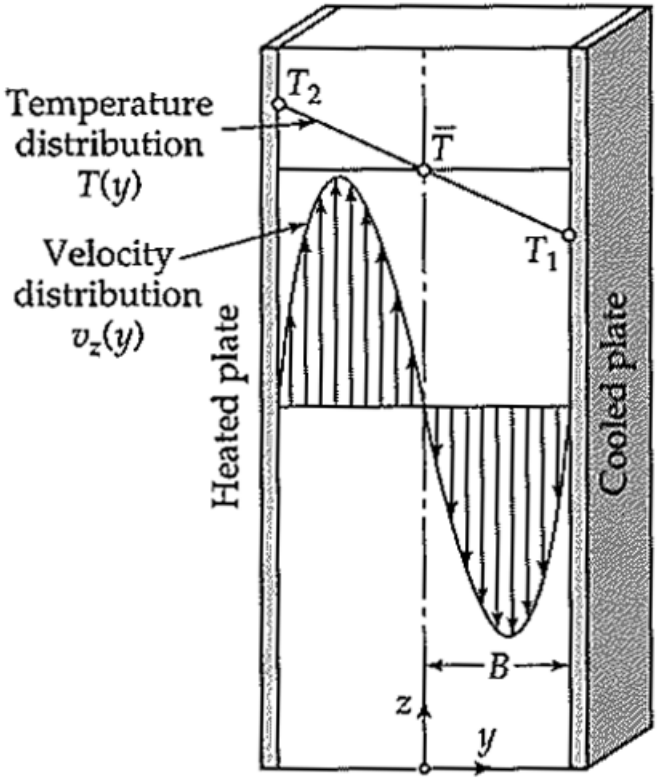
- Understanding of key parameters in heat transfer due to natural convection
- Finding HTC (h) using empirical formulae
- Natural convection in different geometries/configurations

Free/Natural Convection





Fundamental approach

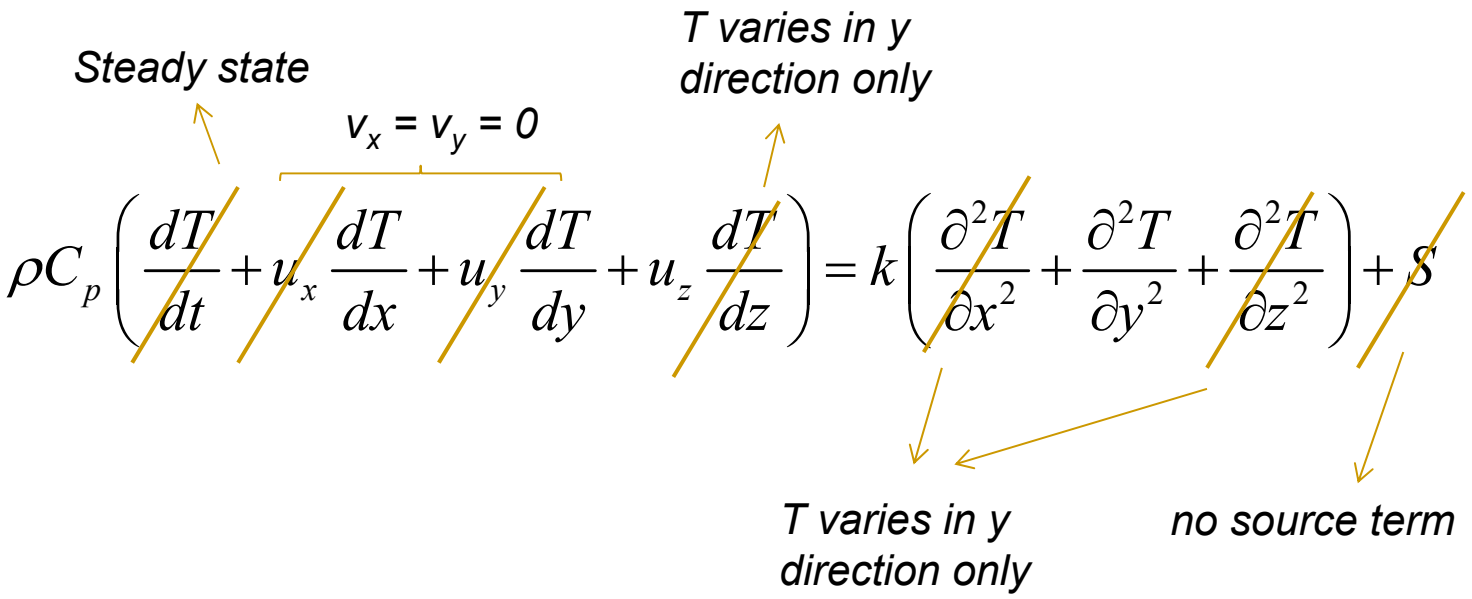


- Fluid between 2 vertical walls with different temperatures
- Fluid near the hot wall rises
- Fluid near the cold wall descends
- System is closed at the top and bottom
  - Continuous circulation of fluid between the plates
  - Upward mass flow rate = Downward mass flow rate
- Tall and wide plates

  → Neglect end effects near the top and bottom

Fundamental approach

- Thermal energy balance
  - Assumptions: steady state, no flow in the x and y direction (i.e. one-directional flow in the z direction only), T varies in the y direction only.



Fundamental approach

- Thermal energy balance: $k\frac{\partial^2 T}{\partial y^2}=0$
- Integrate:
  - First $\partial T/\partial y = C_1$
  - Second $T = C_1 y + C_2$
- Boundary conditions:

$$T\big|_{y=0} = \frac{T_1+T_2}{2} \longrightarrow C_2 = \frac{T_1+T_2}{2} = \overline{T}$$

$$T\big|_{y=B} = T_1 \longrightarrow T_1 = C_1(B) + \frac{T_1+T_2}{2}$$

$$\Rightarrow C_1 = \frac{T_1-T_2}{2B}$$

Fundamental approach

- Temperature distribution:

$$T = \overline{T} - \frac{T_1 - T_2}{2}\frac{y}{B} \qquad \text{OR} \qquad T = \overline{T} - \frac{\Delta T}{2}\frac{y}{B}$$

  - Temperature varies linearly across the gap between the plates

- How much fluid flow is caused by this variation in temperature?
  - Determine from continuity & momentum balances!
  - Assumptions: Steady state, laminar, u_z only ($u_x = u_y = 0$)

$$\cancel{\frac{d\rho}{dt}} + \frac{d(\rho u_x)}{dx} + \cancel{\frac{d(\rho u_y)}{dy}} + \frac{d(\rho u_z)}{dz} = 0$$

ρ is not constant in this case…
→ This is the driving force for the natural convection!

Fundamental approach

From continuity,

$$\frac{d\left(\rho u_z\right)}{dz} = 0 \quad \Rightarrow \quad \rho \frac{du_z}{dz} + u_z \frac{d\rho}{dz} = 0$$

Remember, $T = \overline{T} - \frac{\Delta T}{2} \frac{y}{B}$

- *Variation of T is in y direction only*
- *So, there is no variation of density in the z direction, i.e. $d\rho/dz = 0$*

Therefore, $\frac{du_z}{dz} = 0$

Fundamental approach

Momentum equation for u_z:

- Assume the viscosity of the fluid doesn't change much with temperature

(From continuity eqn.)

$u_x = u_y = 0$

$$\rho \frac{du_z}{dt} = -\rho \left[u_x \frac{du_z}{dx} + u_y \frac{du_z}{dy} + u_z \frac{du_z}{dz} \right] + \mu \left[\frac{d^2 u_z}{dx^2} + \frac{d^2 u_z}{dy^2} + \frac{d^2 u_z}{dz^2} \right] - \frac{dp}{dz} + \rho g_z$$

Steady state

Assumption that u_z varies in the y-direction only

$$\Leftrightarrow \quad \mu \frac{d^2 u_z}{dy^2} = \frac{dp}{dz} - \rho g_z \qquad \Rightarrow \quad \mu \frac{d^2 u_z}{dy^2} = \frac{dp}{dz} + \rho g \qquad (\text{as } g_z = -g)$$

Fundamental approach

Density change with temperature:

$$\rho = \rho\big|_{T=\bar{T}} + \left.\frac{d\rho}{dT}\right|_{T=\bar{T}} \left(T - \bar{T}\right)$$

- Coefficient of thermal expansion:

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad \Rightarrow \frac{\partial \rho}{\partial T} = -\rho\beta$$

- At the reference temperature of $\bar{T}$,

$$\left.\frac{d\rho}{dT}\right|_{T=\bar{T}} = -\bar{\rho}\bar{\beta}$$

ρ & β @ $T = \bar{T}$

So, $\rho = \bar{\rho} - \bar{\rho}\bar{\beta}\left(T - \bar{T}\right)$

For ideal gas,

$$PV = nRT$$

$$\Rightarrow \rho = PM_w / RT$$

$$\Rightarrow \left(\frac{\partial \rho}{\partial T}\right)_P = -\frac{PM_w}{RT^2}$$

$$\beta = -\frac{1}{\rho}\frac{PM_w}{RT^2} = \frac{1}{T}$$

Thermal expansion coeff. for ideal gas

Fundamental approach

- Momentum balance:

$$\mu\frac{d^2u_z}{dy^2}=\frac{dp}{dz}+\rho g \qquad \Rightarrow \mu\frac{d^2u_z}{dy^2}=\frac{dp}{dz}+\bar{\rho}g-\bar{\rho}g\bar{\beta}\left(T-\bar{T}\right)$$

- Variation of T with y,

$$T=\bar{T}-\frac{\Delta T}{2}\frac{y}{B} \quad \Rightarrow T-\bar{T}=-\frac{\Delta T}{2}\frac{y}{B}$$

- So,

$$\mu\frac{d^2u_z}{dy^2}=\frac{dp}{dz}+\bar{\rho}g+\bar{\rho}g\bar{\beta}\frac{\Delta T}{2}\frac{y}{B}$$

Fundamental approach

$$\mu \frac{d^2 u_z}{dy^2} = \frac{dp}{dz} + \bar{\rho} g + \bar{\rho} g \bar{\beta} \frac{\Delta T}{2} \frac{y}{B}$$

- Solution to this eqn:

$$u_z = \frac{\left(\bar{\rho} g \bar{\beta} \Delta T\right) B^2}{12\mu} \left[\left(\frac{y}{B}\right)^3 - \left(\frac{y}{B}\right) \right]$$

$$\Leftrightarrow \; u_z \left(\frac{B\bar{\rho}}{\mu} \right) = \frac{\left(\bar{\rho}^2 g \bar{\beta} \Delta T\right) B^3}{12\mu^2} \left[\left(\frac{y}{B}\right)^3 - \left(\frac{y}{B}\right) \right]$$

Dimensionless

Grashof number, Gr

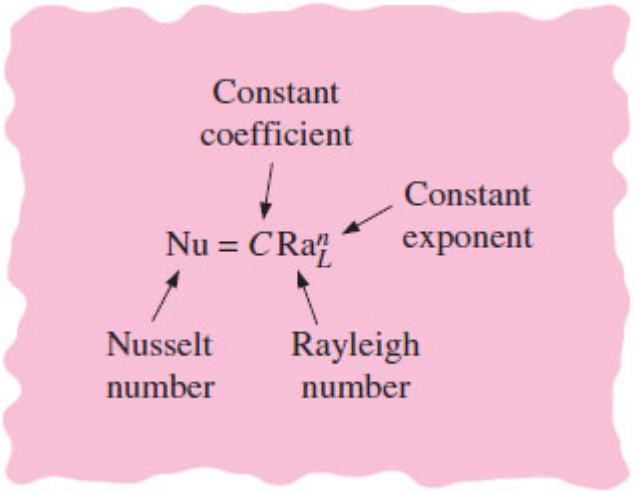
Grashof No

$$Gr = \frac{\left(\bar{\rho}^2 g \bar{\beta} \Delta T\right) B^3}{12 \mu^2} \quad \Rightarrow \quad \mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

where

g = gravitational acceleration, m/s^2
β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C
L_c = characteristic length of the geometry, m
ν = kinematic viscosity of the fluid, m^2/s

Natural convection over surfaces



$$\mathrm{Nu} = \frac{hL_c}{k} = C(\mathrm{Gr}_L\,\mathrm{Pr})^n = C\,\mathrm{Ra}_L^n$$

$$\mathrm{Ra}_L = \mathrm{Gr}_L\,\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\,\mathrm{Pr}$$

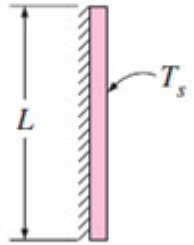
where Ra_L is the **Rayleigh number,** which is the product of the Grashof and Prandtl numbers:

The values of the constants C and n depend on the *geometry* of the surface and the *flow regime,* which is characterized by the range of the Rayleigh number. The value of n is usually $\frac{1}{4}$ for laminar flow and $\frac{1}{3}$ for turbulent flow. The value of the constant C is normally less than 1.

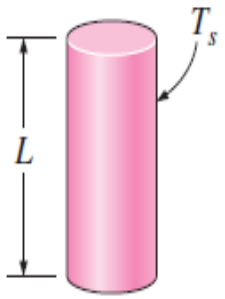
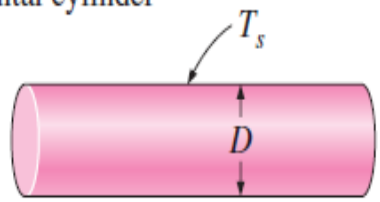
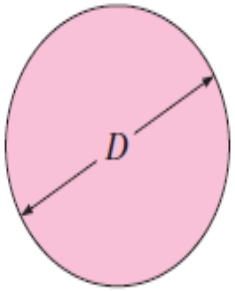
$$L_C = \frac{A_s}{P}$$ = Area / Perimeter

HTC correlations

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$\mathrm{Nu} = 0.59\mathrm{Ra}_L^{1/4}$ $\mathrm{Nu} = 0.1\mathrm{Ra}_L^{1/3}$ $\mathrm{Nu} = \left\{0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)
Horiontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11}	$\mathrm{Nu} = 0.54\mathrm{Ra}_L^{1/4}$ $\mathrm{Nu} = 0.15\mathrm{Ra}_L^{1/3}$
(b) Lower surface of a hot plate (or upper surface of a cold plate) 		10^5–10^{11}	$\mathrm{Nu} = 0.27\mathrm{Ra}_L^{1/4}$

HTC correlations

Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{\mathrm{Gr}_L^{1/4}}$
Horizontal cylinder 	D	$\mathrm{Ra}_D \leq 10^{12}$	$\mathrm{Nu} = \left\{0.6 + \frac{0.387\mathrm{Ra}_D^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$
Sphere 	D	$\mathrm{Ra}_D \leq 10^{11}$ ($\mathrm{Pr} \geq 0.7$)	$\mathrm{Nu} = 2 + \frac{0.589\mathrm{Ra}_D^{1/4}}{[1 + (0.469/\mathrm{Pr})^{9/16}]^{4/9}}$

From the lab…

- Thermal convection from a localised heat source

Example 1 (From lect. 5-2, ex. 2)

A flat wall of fire-clay brick (0.5 m thick) initially has a uniform temperature of 200K. One of its faces is suddenly exposed to hot gas at 1200K. ~~If the heat transfer coefficient on the hot side is 7.38 W/m2.K~~ and the other face of the wall is insulated, determine:

- The time it takes for the centre of the wall to reach 600K

Properties of fire-clay brick:

k = 1.125 W/m.K;

Cp = 919 J/kg.K;

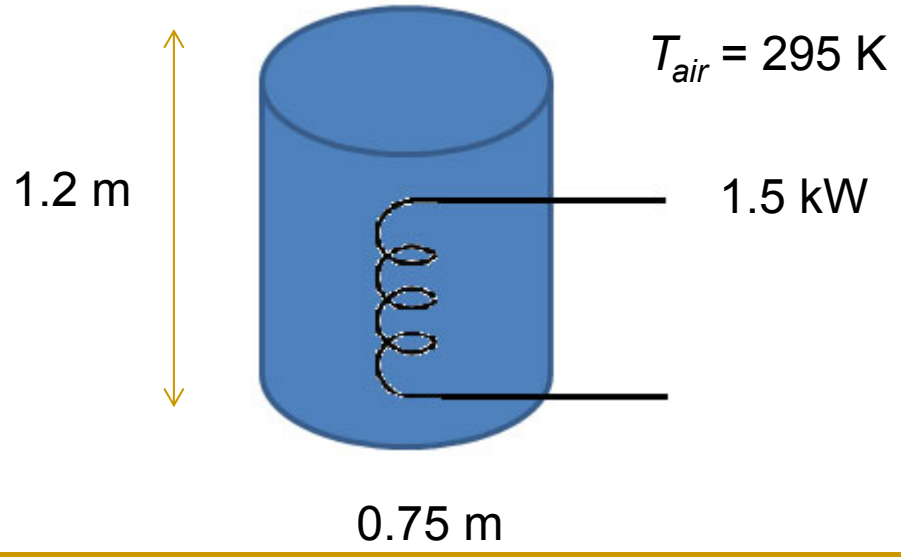
ρ = 2310 kg/m^3

Estimate the heat transfer coefficient, assuming that it stays constant throughout the transient stage!

Does the number quoted above make sense?

Example 2

Determine the surface temperature of a cylindrical tank (0.75 m Diameter, 1.2m high), which contains a transformer immersed in an oil bath, producing a uniform surface temperature condition. All heat loss from the surface may be assumed to be due to natural convection to surrounding air at 295 K. The heat dissipation rate from the transformer is constant at 1.5 kW.



[Welty et al. (2001)]

Further questions

- Does natural convection also occur in cases where forced convection occurs?
  - Yes, generally mixed convection heat transfer is correlated through combinations of the 2 phenomena:

$$Nu^n = Nu_F^n \pm Nu_N^n$$

The + or – sign depends if the buoyancy works in the same, opposite, or transverse direction to the direction of the forced convection

  - The effect of free convection on heat transfer is negligible to that of forced convection if the liquid flow velocities are:

$$u_{free\ convection} < u_{forced\ convection}$$

  - So:
    - $Gr \gg Re^2$ → natural convection is dominant
    - $Gr \approx Re^2$ → combined forced & free convection
    - $Gr \ll Re^2$ → forced convection dominant

Key points

- Key dimensionless parameter: Grashof number
- Finding “h” using empirical formulae
- Natural convection in different geometries & configurations

Reference

- Cengel, Heat and Mass Transfer: A Practical Approach, 3 ed., 2006 (main reference)
- Incropera et al, Fundamentals of Heat and Mass transfer, 6 ed. 2007
- Geankoplis, C.J., Transport Processes and Separation Process Principles, 4th ed, 2003
- McCabe et al, Unit Operations of Chemical Engineering, 7 ed., 2005
- Griskey, R.G., Transport Phenomena and Unit Operations: A Combined Approach, 2002, John Wiley and Sons, Inc.