# Natural convection

## 1 Errata

- Slide 9 reverses the wall temperatures in the first temperature-profile expression. With $T(-B) = T_2$, $T(B) = T_1$ and $T_2 > T_1$, use

$$T(y) = \frac{T_1 + T_2}{2} - \frac{T_2 - T_1}{2B} y$$

- Slide 15 includes a factor of 12 in the first expression for the Grashof number. Use the conventional definition

$$\mathrm{Gr} = \frac{g\beta\Delta T B^3}{\nu^2}$$

  The factor 12 in the slide's first expression belongs to the slot velocity solution.

- Slide 17 extends the hot-downward or cold-upward plate correlation $\overline{\mathrm{Nu}} = 0.27\mathrm{Ra}^{1/4}$ to $10^{11}$. Use the stated textbook range $10^5 \leq \mathrm{Ra} \leq 10^{10}$ in these notes. The convection-model reference gives the same upper limit. Do not extrapolate this relation to $10^{11}$.

## 2 Reading list

- Çengel and Ghajar (2015), chapter 9, and Çengel (2006), chapter 9, for natural-convection correlations.
- Incropera et al. (2007), chapter 9, for boundary layers and enclosures.
- Bird et al. (2006), section 10.9, for the closed-slot derivation.
- Welty et al. (2001) for the transformer-tank problem.
- Geankoplis (2003), McCabe et al. (2005) and Griskey (2002) for further reading.

## 3 Choose a buoyancy model

| Given conditions | Model and checks |
|---|---|
| Still surrounding fluid; vertical wall | Use height and a vertical-plate relation after calculating film properties and Rayleigh number. |
| Horizontal plate | Use area/perimeter for one face; select hot-up/cold-down or hot-down/cold-up orientation. |
| Horizontal cylinder or sphere | Use diameter and that geometry's correlation; the sphere retains Nu = 2 as buoyancy vanishes. |
| Imposed flow is also present | Compare $\mathrm{Gr}/\mathrm{Re}^2$ before treating the problem as purely forced or purely natural convection. |

## 4 Buoyancy-driven flow

DEFINITION
**Natural convection**

Convection driven by buoyancy arising from fluid density differences in a gravitational field. Temperature or composition differences can create the density differences.

QUIZ
**1. The source of motion**
Air beside a hot vertical wall rises without a fan. What drives this motion?

1. A prescribed free-stream velocity.
2. Warm air is less dense than the surrounding air, so it rises under gravity.
3. Thermal conductivity acting as a mechanical force.
4. Radiation pressure in the classroom model.

See solution for Quiz 1

## 5 Fundamental approach

### 5.1 Geometry and assumptions

Consider two tall, wide vertical plates separated by $2B$. Let $y$ measure distance across the gap from its midplane and $z$ point upward. The hot wall at $y = -B$ is at $T_2$; the cold wall at $y = B$ is at $T_1$, with $T_2 > T_1$. The top and bottom are closed. Fluid rises on the hot side, turns at the top, descends on the cold side and turns at the bottom. The interior model neglects

these turning regions and width-edge effects. An aspect ratio above roughly ten is the source's guideline, not a guarantee for every fluid or temperature difference. Cavity windows motivate this geometry, but whole-cavity heat transfer requires the end circulation too.

Take steady laminar flow, $u_x = u_y = 0$, $u_z = u_z(y)$ and $T = T(y)$. Fluid conductivity, viscosity and heat capacity are constant at the mean wall temperature. Neglect volumetric heating and viscous dissipation. Density variations are retained in gravity and linearised about the reference state, the Boussinesq approximation. Its validity requires a small fractional density change. It cannot describe arbitrary large temperature excursions.

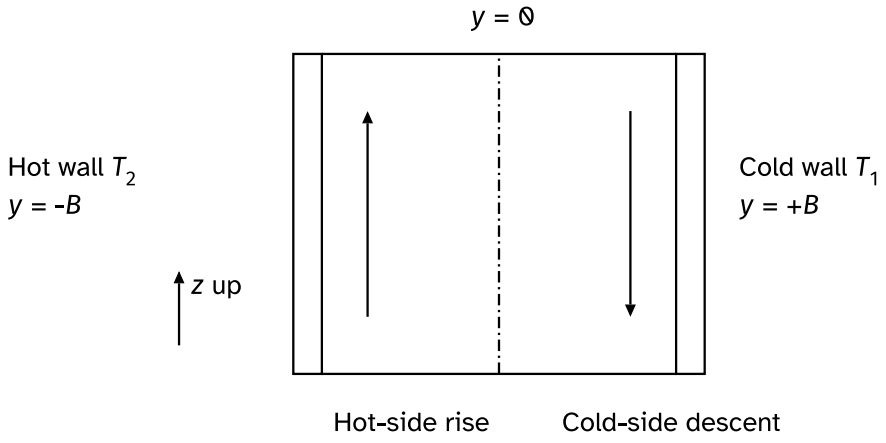



Figure 1: Closed vertical slot with the hot wall on the left and the cold wall on the right. Arrows show the opposite directions of interior flow.

Use $u_z$ in $\mathrm{m\,s^{-1}}$, $B, y, z$ in m, pressure $p$ in Pa, density $\rho$ in $\mathrm{kg\,m^{-3}}$, dynamic viscosity $\mu$ in $\mathrm{Pa\,s}$, kinematic viscosity $\nu = \mu/\rho$ in $\mathrm{m^2\,s^{-1}}$, acceleration $g$ in $\mathrm{m\,s^{-2}}$, conductivity $k$ in $\mathrm{W\,m^{-1}\,K^{-1}}$ and specific heat $C_p$ in $\mathrm{J\,kg^{-1}\,K^{-1}}$. Heat rate is $\dot{Q}$ in W and heat flux is $q$ in $\mathrm{W\,m^{-2}}$, as in the earlier topics. The source uses $Q$ for a rate. $\bar{T}$ and $\bar{\rho}$ denote reference temperature and density, not the internal-flow mixing-cup means of topic 4.

## 5.2 Thermal energy balance and integrations

The constant-property energy equation is

$$\rho C_p \left( \frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T \right) = k \nabla^2 T + S_e \tag{1}$$

Steadiness removes the time derivative. Zero transverse velocity removes $x$ and $y$ advection. Vertical advection vanishes because $\partial T/\partial z = 0$, although $u_z$ is nonzero. Only the $y$ conduction derivative survives, and the source is zero. Thus Eq. 1 becomes

$$\frac{d^2T}{dy^2} = 0, \qquad \frac{dT}{dy} = C_1, \qquad T = C_1 y + C_2 \tag{2}$$

Add the wall conditions $T(-B) = T_2$ and $T(B) = T_1$ to find $C_2$. Subtract them to find $C_1$:

$$\bar{T} = \frac{T_1 + T_2}{2}, \qquad C_2 = \bar{T}, \qquad C_1 = \frac{T_1 - T_2}{2B} \tag{3}$$

The midplane temperature is a consequence of linearity and both wall conditions. It is not an extra imposed condition. Define $\Delta T = T_2 - T_1 > 0$ in this slot. Then

$$T(y) = \bar{T} - \frac{\Delta T}{2}\frac{y}{B} \tag{4}$$

Equation Eq. 4 returns the correct temperature at each wall and falls from left to right. Heat crosses the gap by conduction even though fluid moves along its isotherms. The special geometry decouples energy from momentum. Near the turns a two-dimensional, coupled calculation is necessary.

### 5.3 Continuity

With steady flow and zero transverse velocities, mass conservation reduces to

$$\frac{\partial(\rho u_z)}{\partial z} = 0.$$

Expanding gives

$$\rho\frac{\partial u_z}{\partial z} + u_z\frac{\partial \rho}{\partial z} = 0 \tag{5}$$

Density depends on $T(y)$, so $\partial\rho/\partial z = 0$. Equation Eq. 5 therefore gives $\partial u_z/\partial z = 0$, the developed velocity assumption. Density varies across space but a vertically moving particle stays on the same isotherm. This is compatible with incompressible Boussinesq continuity. Setting density uniform inside the gravity term, however, would remove the driving buoyancy.

### 5.4 Momentum, density and expansion

The vertical momentum equation, before cancellation, is

$$\begin{aligned} \rho\left(\frac{\partial u_z}{\partial t} + \boldsymbol{u}\cdot\nabla u_z\right) &= -\frac{\partial p}{\partial z} \\ &\quad +\mu\nabla^2 u_z + \rho g_z \\ g_z &= -g \end{aligned} \tag{6}$$

All inertia vanishes: the flow is steady, transverse velocities are zero and continuity removed the vertical derivative. Only $y$ viscous diffusion remains. Constant $\mu$ is an approximation because real viscosity changes with temperature. The remaining balance is

$$\mu\frac{d^2 u_z}{dy^2} = \frac{dp}{dz} + \rho g \tag{7}$$

Viscous shear balances pressure gradient and weight. For uniform density and a hydrostatic gradient the right side would be zero. Linearise density at constant pressure about $\bar{T}$ and define thermal expansion $\beta$ in $\mathrm{K}^{-1}$:

$$\begin{aligned} \beta &= -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p \\ \rho(T) &\simeq \bar{\rho}\left[1 - \bar{\beta}(T - \bar{T})\right] \end{aligned} \tag{8}$$

The derivative at the reference state is $-\bar{\rho}\bar{\beta}$. For an ideal gas $\rho = pM_w/(R_gT)$, where molar mass $M_w$ is in $\mathrm{kg\,mol^{-1}}$ and $R_g$ is the universal gas constant in $\mathrm{J\,mol^{-1}\,K^{-1}}$:

$$\left(\frac{\partial \rho}{\partial T}\right)_p = -\frac{pM_w}{R_gT^2}, \qquad \beta = \frac{1}{T} \tag{9}$$

Absolute kelvin temperature is required in Eq. 9. For liquids, obtain $\beta$ from property data. Water near room temperature expands much less per degree than air and below its maximum-density temperature can have negative $\beta$. A correlation assuming ordinary positive expansion needs reconsideration in that regime.

Substitute Eq. 8 and Eq. 4 into Eq. 7:

$$\begin{aligned} \mu\frac{d^2u_z}{dy^2} &= \frac{dp}{dz} + \bar{\rho}g - \bar{\rho}g\bar{\beta}(T-\bar{T}) \\ &= P_0 + \frac{\bar{\rho}g\bar{\beta}\Delta T}{2B}y \\ P_0 &= \frac{dp}{dz} + \bar{\rho}g \end{aligned} \tag{10}$$

$P_0$ is a pressure-gradient combination in $\mathrm{Pa\,m^{-1}}$, not perimeter. Two integrations of Eq. 10 give

$$\mu u_z = \frac{P_0y^2}{2} + \frac{\bar{\rho}g\bar{\beta}\Delta T}{12B}y^3 + C_3y + C_4 \tag{11}$$

No slip at both walls gives $C_4 = -P_0B^2/2$ and

$$C_3 = -\frac{\bar{\rho}g\bar{\beta}\Delta TB}{12}.$$

Closure requires zero net volume flow in the Boussinesq model. Thus

$$\begin{aligned} 0 &= \int_{-B}^{B} u_z\,dy = -\frac{2P_0B^3}{3\mu} \\ P_0 &= 0, \qquad \frac{dp}{dz} = -\bar{\rho}g \end{aligned} \tag{12}$$

The odd cubic and linear terms integrate to zero, leaving only the even pressure term. This is zero volume flow at reference density, hence zero mass flow to the order retained by Boussinesq. Keeping the tiny product of density perturbation and velocity would introduce a higher-order correction. The result is

$$u_z = \frac{g\bar{\beta}\Delta TB^2}{12\bar{\nu}}\left[\left(\frac{y}{B}\right)^3 - \frac{y}{B}\right] \tag{13}$$

DEFINITION
**Grashof number**

The dimensionless thermal-buoyancy group

$$\mathrm{Gr}_L = \frac{g\beta|\Delta T|L^3}{\nu^2}.$$

It measures buoyancy relative to the viscous scale, with thermal expansion coefficient $\beta$, kinematic viscosity $\nu$ and the geometry's specified length $L$.

Small Gr means weak buoyancy on this viscous scale; large Gr means stronger buoyancy-driven motion is possible. It does not by itself guarantee turbulence or enhanced heat transfer. The slot solution still transfers heat across its gap by conduction. Here use half-gap $B$; the external-surface example below uses its correlation length. Doubling that length multiplies Gr by eight. Substituting full gap for half-gap would therefore give the wrong velocity scale. Composition differences can also drive buoyancy, as shown later in the mass-transfer topic.

Equation Eq. 13 is zero at both walls and at the midplane, positive on the hot side and negative on the cold side. Differentiating the bracket gives extrema at $y/B = \pm 1/\sqrt{3}$, with maximum speed

$$\frac{g\bar{\beta}\Delta T B^2}{18\sqrt{3}\,\bar{\nu}}.$$

Multiplication by $B/\bar{\nu}$ yields the dimensionless form

$$\frac{u_z B}{\bar{\nu}} = \frac{\mathrm{Gr}_B}{12}\left[\left(\frac{y}{B}\right)^3 - \frac{y}{B}\right]$$

The Grashof number based on half-gap is

$$\mathrm{Gr}_B = \frac{g\bar{\beta}\Delta T B^3}{\bar{\nu}^2} \tag{14}$$

The velocity in this viscous closed slot produces a Reynolds scale proportional to Gr. The factor twelve belongs to this particular solution. Source slide 15′s left-hand expression includes it inside Gr; use the conventional definition below. In an inertia-buoyancy comparison the appropriate counterpart is $\mathrm{Re}^2$. These two balances should not be confused.

QUIZ

**2. Circulation in a closed slot**

In the stated closed-slot Boussinesq model, net volume flow through a horizontal section is zero. What does this imply about local motion?

1. Velocity is zero everywhere.
2. Both sides flow upward at the same speed.
3. Heat transfer is zero across the slot.
4. Upward flow on the hot side balances downward flow on the cold side.

See solution for Quiz 2

### 5.5 Example: calculating the Grashof number

For external natural convection,

$$\mathrm{Gr}_{L_c} = \frac{g\beta|T_s - T_\infty|L_c^3}{\nu^2} \tag{15}$$

For $L_c = 0.3\,\mathrm{m}$, air 30 K hotter than 300 K, $T_f = 315\,\mathrm{K}$, $\beta = 1/T_f$ and $\nu = 1.7 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$, substitution gives $\mathrm{Gr}_{L_c} = 8.73 \times 10^7$.

## 6 Natural convection over surfaces

DEFINITION

**Rayleigh number**

The thermal natural-convection group

$$\mathrm{Ra}_L = \mathrm{Gr}_L\mathrm{Pr} = \frac{g\beta|\Delta T|L^3}{\nu\alpha},$$

using the same length and properties in both factors. It measures buoyancy relative to momentum and thermal diffusion.

At small Ra, diffusion suppresses the thermal contrasts and motion needed for strong convective enhancement. Larger Ra can support stronger convection, but onset and transition depend on geometry and orientation. In an unstable layer heated from below an onset criterion may apply; it is not a universal criterion for every wall. The example above gives $\mathrm{Gr} = 8.73 \times 10^7$; with $\mathrm{Pr} = 0.70$, $\mathrm{Ra} = 6.11 \times 10^7$. Use that value to check the chosen surface correlation's range, rather than inserting Gr in place of Ra or using a length from another geometry.

Use the established definitions $\mathrm{Pr} = \nu/\alpha, \alpha = k/(\rho C_p)$ and $\mathrm{Nu} = hL_c/k$. Introduce Rayleigh:

$$\mathrm{Ra}_{L_c} = \mathrm{Gr}_{L_c}\mathrm{Pr} = \frac{g\beta\Delta T L_c^3}{\nu\alpha} \tag{16}$$

Viscosity spreads momentum while thermal diffusion reduces the temperature contrast producing buoyancy. Both enter Eq. 16. Many average correlations have

$$\overline{\mathrm{Nu}} = C\mathrm{Ra}_{L_c}^{n}$$

Converting Nusselt number to a coefficient gives

$$\bar{h} = \frac{k}{L_c} C\mathrm{Ra}_{L_c}^{n} \tag{17}$$

For fixed properties, $h \propto \Delta T^{n} L_c^{3n-1}$. Quarter-power laminar forms give $h \propto L_c^{-1/4}$; one-third-power turbulent forms cancel length. Larger area then increases heat loss without changing $h$. The source's $L_c = A_s/P$ applies only to horizontal plates. Here $A_s$ is exposed area in $\mathrm{m}^2$ and $P$ its perimeter in m; $p$ remains pressure.

QUIZ

**3. Rayleigh-number scaling**

At fixed properties and temperature difference, the characteristic length doubles. How does $\mathrm{Ra}_{L_c}$ change?

1. It increases by a factor of eight.
2. It doubles.
3. It increases by a factor of four.
4. It stays the same.

See solution for Quiz 3

## 6.1 Correlations used in the worked examples

Evaluate fluid properties at

$$T_f = \frac{T_s + T_\infty}{2}$$

and calculate $\mathrm{Ra} = \mathrm{Gr}\,\mathrm{Pr}$. For the vertical-plate examples, use height $L$ and

$$\overline{\mathrm{Nu}}_L = 0.59\mathrm{Ra}_L^{1/4}, \quad 10^4 \le \mathrm{Ra}_L \le 10^9 \tag{18}$$

or the full-range Churchill-Chu form,

$$\overline{\mathrm{Nu}}_L = \left[0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{\left[1 + \left(\frac{0.492}{\mathrm{Pr}}\right)^{9/16}\right]^{8/27}}\right]^2 \tag{19}$$

The full-range fit is conventionally used up to $\mathrm{Ra}_L \sim 10^{12}$. A vertical cylinder can use the plate result when

$$D \ge \frac{35L}{\mathrm{Gr}_L^{1/4}} \tag{20}$$

For $L = 1\ \mathrm{m}$ and $\mathrm{Gr}_L = 10^9$, this requires $D \ge 0.197\ \mathrm{m}$.

The additional horizontal-cylinder problem uses

$$\overline{\mathrm{Nu}}_D = \left[0.60 + \frac{0.387\mathrm{Ra}_D^{1/6}}{\left[1 + \left(\frac{0.559}{\mathrm{Pr}}\right)^{9/16}\right]^{8/27}}\right]^2, \qquad \mathrm{Ra}_D \le 10^{12} \tag{21}$$

## 6.2 Horizontal plates and spheres

These are average coefficients for isolated isothermal surfaces in a large, still fluid. Evaluate fluid properties at $T_f = (T_s + T_\infty)/2$, with absolute temperature for $\beta \simeq 1/T_f$ in an ideal gas. Use $|T_s - T_\infty|$ in Rayleigh number; orientation determines which relation applies.

| Geometry and orientation | Length in Nu and Ra | Average correlation | Range |
|---|---|---|---|
| Vertical plate | Height $L$ | Eq. 18 or Eq. 19 | Power law: $10^4 \le \mathrm{Ra}_L \le 10^9$; Churchill-Chu: up to about $10^{12}$ |
| Vertical cylinder approximated as a plate | Height $L$ | Plate correlation | Also satisfy Eq. 20 |
| Horizontal cylinder | Diameter $D$ | Eq. 21 | $\mathrm{Ra}_D \le 10^{12}$ |
| Hot face up, or cold face down | $L_c = A_s/P$ for that face | $\overline{\mathrm{Nu}} = 0.54\mathrm{Ra}^{1/4}$ | $10^4 \le \mathrm{Ra} \le 10^7$ |
| Hot face up, or cold face down | $L_c = A_s/P$ | $\overline{\mathrm{Nu}} = 0.15\mathrm{Ra}^{1/3}$ | $10^7 \le \mathrm{Ra} \le 10^{11}$ |
| Hot face down, or cold face up | $L_c = A_s/P$ | $\overline{\mathrm{Nu}} = 0.27\mathrm{Ra}^{1/4}$ | $10^5 \le \mathrm{Ra} \le 10^{10}$ |
| Sphere | Diameter $D$ | Eq. 22 below | $\mathrm{Pr} \ge 0.7$, $\mathrm{Ra}_D \le 10^{11}$ |

The plate relations and orientation rules follow Çengel and Ghajar (2015) and the Ansys convection-model documentation. Do not combine the plate area from both faces when calculating the length for one face. Evaluate each exposed face separately, then add its heat rate.

For a sphere, the Churchill estimate is

$$\overline{\mathrm{Nu}}_D = 2 + \frac{0.589\mathrm{Ra}_D^{1/4}}{\left[1 + \left(\frac{0.469}{\mathrm{Pr}}\right)^{9/16}\right]^{4/9}} \tag{22}$$

Use $\bar{h} = \overline{\mathrm{Nu}}_D k/D$. The diameter basis and range are also given in the EES sphere model. As buoyancy tends to zero this tends to the stagnant-sphere limit $\mathrm{Nu}_D = 2$.

For example, at $\mathrm{Ra} = 10^6$ both laminar plate relations apply. They give $\overline{\mathrm{Nu}} = 17.1$ for a hot upward-facing surface and 8.54 for a hot downward-facing surface. For equal $k$ and $L_c$, the upward-facing coefficient is twice as large. The different temperature and orientation conditions are essential to this comparison.

For a horizontal plate, $L_c = A_s/P$. A hot upward-facing plate lets warm fluid rise away; a hot downward-facing plate traps it against the surface.

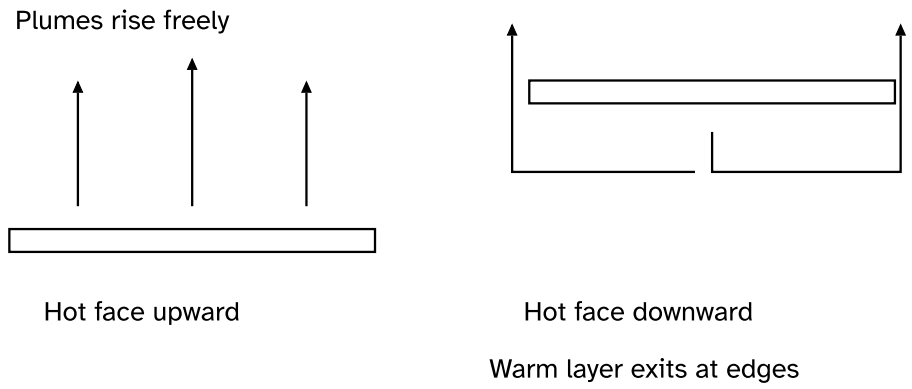



Figure 2: Comparison of buoyant flow above and below a hot horizontal plate.

Buoyant motion above and below a hot horizontal surface.

QUIZ

**4. Horizontal plate orientation**

Two identical isothermal hot horizontal plates face still cooler air. One hot surface faces upward and the other downward. Within the stated laminar correlation ranges, which orientation transfers heat more readily by natural convection?

1. Both must have the same coefficient because their areas match.
2. The downward-facing surface, because heated air becomes denser.
3. The upward-facing surface, because warmed fluid can rise away from it.
4. Neither can transfer heat without a fan.

See solution for Quiz 4

## 6.3 Example: observing a heated plume

A localised heater produces a thin heated layer, a rising plume and a broader return flow. The return flow carries the plume's mass downward. In the closed slot the return is beside the plume; in an open room it spreads farther away.

## 7 Example 1: Fire-clay brick wall

EXERCISE
**Exercise 1. Heating a one-sided brick wall**

A flat wall of fire-clay brick (0.5 m thick) initially has a uniform temperature of 200K. One of its faces is suddenly exposed to hot gas at 1200K. If the heat transfer coefficient on the hot side is $7.38\ \mathrm{W/m^2.K}$ and the other face of the wall is insulated, determine:

- The time it takes for the centre of the wall to reach 600K

Properties of fire-clay brick: $k = 1.125\ \mathrm{W/m.K}$; $C_p = 919\ \mathrm{J/kg.K}$; $\rho = 2310\ \mathrm{kg/m^3}$

Estimate the heat transfer coefficient, assuming that it stays constant throughout the transient stage! Does the number quoted above make sense?

See solution for Exercise 1

## 8 Example 2: Transformer tank

EXERCISE
**Exercise 2. Surface temperature of a transformer tank**

Determine the surface temperature of a cylindrical tank (0.75 m Diameter, 1.2m high), which contains a transformer immersed in an oil bath, producing a uniform surface temperature condition. All heat loss from the surface may be assumed to be due to natural convection to surrounding air at 295 K. The heat dissipation rate from the transformer is constant at 1.5 kW.

(Welty et al. 2001)

See solution for Exercise 2

## 9 Further questions

Buoyancy persists when a fan is switched on. Compare it with forced inertia using the same characteristic length and properties:

$$\frac{\mathrm{Gr}}{\mathrm{Re}^2} = \frac{g\beta\Delta T L_c}{u_\infty^2} \tag{31}$$

A buoyant inertial velocity scale is $u_N \sim \sqrt{g\beta\Delta T L_c}$, so Eq. 31 is the square of $u_N/u_\infty$. Below about 0.1 forced convection dominates; above about ten natural convection dominates. Between these limits mixed convection matters. These thresholds are rules of thumb and depend on geometry. For the 0.3 m, 30 K plate illustration, the ratios 10 and 0.1 correspond to imposed speeds about 0.17 and 1.67 m $\mathrm{s^{-1}}$.

An empirical combination sometimes used is

$$\mathrm{Nu}^{n_m} = \mathrm{Nu}_F^{n_m} \pm \mathrm{Nu}_N^{n_m}, \qquad n_m \simeq 3 \tag{32}$$

Here $n_m$ is a fitted blending exponent, distinct from the quarter- or third-power exponent in Eq. 17. Subscripts F and N indicate forced and natural convection. The plus sign describes assisting and commonly transverse flows; opposing flow uses a geometry-specific subtractive form. A fan blowing downward beside a hot vertical wall can oppose the upward buoyant layer. The simple minus expression must have a nonnegative right side and cannot be extrapolated through cancellation or reversal. It is not a universal prediction for all mixed flows.

QUIZ

**5. Mixed convection**

For a hot vertical wall with imposed airflow, $\mathrm{Gr}/\mathrm{Re}^2 = 1$ using a common length and properties. What does the stated screening rule suggest?

1. Ignore buoyancy entirely.
2. Both the imposed airflow and buoyancy may affect heat transfer.
3. Ignore the imposed flow entirely.
4. Heat transfer must be zero.

See solution for Quiz 5

## 9.1 Deriving a radiation coefficient

Factor the radiation temperature difference:

$$T_s^4 - T_{\mathrm{surr}}^4 = (T_s - T_{\mathrm{surr}})(T_s + T_{\mathrm{surr}})(T_s^2 + T_{\mathrm{surr}}^2).$$

Writing $q_{\mathrm{rad}} = h_r(T_s - T_{\mathrm{surr}})$ gives

$$h_r = \epsilon\sigma(T_s^2 + T_{\mathrm{surr}}^2)(T_s + T_{\mathrm{surr}}) \tag{33}$$

Use kelvin temperatures. Add $h + h_r$ only when convection and radiation have the same reference temperature; otherwise add their heat rates.

QUIZ

**6. Convection and radiation together**

A hot surface loses heat to air at $T_\infty$ and radiates to surroundings at a different temperature $T_{\mathrm{surr}}$. How should the two heat paths be combined?

1. Add convection and radiation resistances in series.
2. Use the larger coefficient and discard the other path.
3. Always use $(h + h_r)A(T_s - T_\infty)$.
4. Calculate each heat rate with its own reference temperature, then add the rates.

See solution for Quiz 6

## 10 Additional examples

EXERCISE

**Exercise 3. Cooling a horizontal metal cylinder in still air**

A long horizontal metal cylinder of diameter $0.180\,\mathrm{m}$ starts uniformly at $180\,^{\circ}\mathrm{C}$ in still air at $30\,^{\circ}\mathrm{C}$. Use solid properties $k_w = 45\,\mathrm{W\,m^{-1}\,K^{-1}}$, $\rho = 7800\,\mathrm{kg\,m^{-3}}$ and $C_p = 500\,\mathrm{J\,kg^{-1}\,K^{-1}}$. At the initial film temperature, use air properties $k = 0.032\,\mathrm{W\,m^{-1}\,K^{-1}}$, $\nu = 2.40 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$ and $\mathrm{Pr} = 0.70$.

Estimate the natural-convection coefficient and the centre temperature after 20 minutes. Neglect end effects and radiation, and hold the estimated coefficient and properties constant over the cooling interval. If a fan instead produces a coefficient of $30\,\mathrm{W\,m^{-2}\,K^{-1}}$, can the same transient model still be used?

See solution for Exercise 3

## 11 What you should now be able to calculate

Choose a correlation from geometry and orientation, calculate a convection coefficient or unknown surface temperature, and check whether radiation or mixed convection matters.

Practice: Workshop 7 for natural convection from a horizontal pipe.

## 12 Report an error

Email Ranjeet Utikar with the title Natural convection, the URL `/heat-transport/natural-convection/`, and the equation or passage to check.

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# Solutions

## Quiz solutions

### Quiz 1. The source of motion

Correct choice: 2.

Air next to the hot wall warms and becomes less dense. It rises through the cooler, denser air under gravity. This buoyancy drives the flow; no fan is needed.

Back to Quiz 1

### Quiz 2. Circulation in a closed slot

Correct choice: 4.

The volume moving upward beside the hot wall equals the volume moving downward beside the cold wall. The net flow through a horizontal section is zero even though the fluid circulates. In the developed interior of this model, heat crosses the gap by conduction.

Back to Quiz 2

### Quiz 3. Rayleigh-number scaling

Correct choice: 1.

Rayleigh number depends on the cube of the characteristic length:

$$\mathrm{Ra}_{L_c} = \frac{g\beta\Delta T L_c^3}{\nu\alpha}$$

With properties and temperature difference unchanged,

$$\frac{\mathrm{Ra}_{2L_c}}{\mathrm{Ra}_{L_c}} = 2^3 = 8$$

Rayleigh number increases by a factor of eight. The heat transfer coefficient must then be found from the appropriate correlation.

Back to Quiz 3

### Quiz 4. Horizontal plate orientation

Correct choice: 3.

Warm, lighter fluid can leave an upward-facing hot surface directly. Beneath a downward-facing hot surface, the warm layer is stably trapped and must move toward an edge. The stated correlations therefore give the upward-facing surface the larger coefficient.

Back to Quiz 4

### Quiz 5. Mixed convection

Correct choice: 2.

A ratio near one means that buoyancy and the imposed airflow can both affect heat transfer. Include both effects when choosing a model. Their directions matter because the fan can assist or oppose the rising warm air.

Back to Quiz 5

### Quiz 6. Convection and radiation together

Correct choice: 4.

Calculate convection using the air temperature:

$$\dot{Q}_{\text{conv}} = hA(T_s - T_\infty)$$

Calculate linearised radiation using the surrounding surface temperature:

$$\dot{Q}_{\text{rad}} = h_r A(T_s - T_{\text{surr}})$$

Then add the rates:

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

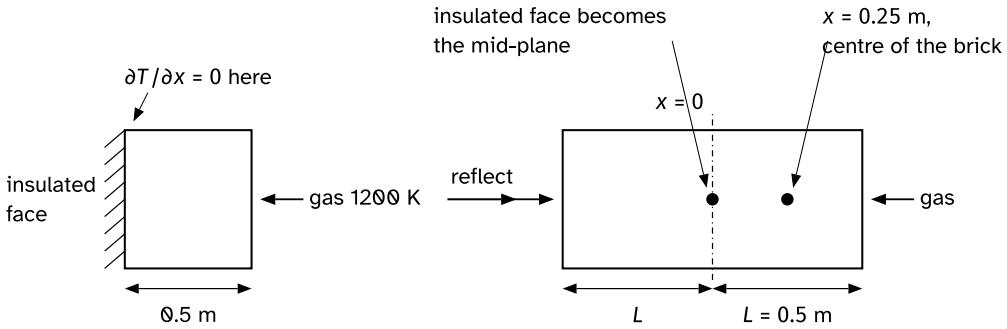
You can combine the coefficients with one temperature difference only when the air and surrounding surfaces have the same temperature.

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## Exercise solutions

### Exercise 1. Heating a one-sided brick wall

Reflecting the wall across its insulated face allows us to use the symmetric plane-wall solution. The half-thickness labelled $L$ in the schematic is denoted $b$ below, to distinguish it from the vertical convection length.




**Data**

| Quantity | Value |
|---|---|
| Actual wall thickness | $b = 0.50\ \mathrm{m}$ |
| Initial brick temperature | $T_i = 200\ \mathrm{K}$ |
| Hot-gas temperature | $T_\infty = 1200\ \mathrm{K}$ |
| Target temperature | $T = 600\ \mathrm{K}$ |
| Brick conductivity | $k_w = 1.125\ \mathrm{W\,m^{-1}\,K^{-1}}$ |
| Brick density | $\rho_w = 2310\ \mathrm{kg\,m^{-3}}$ |
| Brick specific heat capacity | $C_{p,w} = 919\ \mathrm{J\,kg^{-1}\,K^{-1}}$ |
| Specified transient coefficient | $h = 7.38\ \mathrm{W\,m^{-2}\,K^{-1}}$ |
| Air conductivity at 700 K | $k = 0.0530\ \mathrm{W\,m^{-1}\,K^{-1}}$ |
| Air kinematic viscosity at 700 K | $\nu = 6.58 \times 10^{-5}\ \mathrm{m^2\,s^{-1}}$ |
| Air Prandtl number at 700 K | $\mathrm{Pr} = 0.697$ |

The lecturer's initial-film air properties allow an independent check of the specified coefficient. The slide supplies no wall height for this convection estimate.

**Assumptions**

- One-dimensional transient conduction through the wall.
- Constant solid properties and heat transfer coefficient.
- No internal heat generation.
- Heat transfer through the edges is neglected.

- Radiation is neglected in the classroom model.
- A vertical panel of height $L = 1.0\ \mathrm{m}$ is used only to estimate the convection coefficient.

**Calculations**

First check the supplied coefficient. Initially the exposed wall is at 200 K, giving

$$T_f = \frac{1200 + 200}{2} = 700\ \mathrm{K}$$

and $\beta = 1/T_f$ for ideal-gas air. The vertical-plate Rayleigh number is

$$\mathrm{Ra}_L = \frac{g\beta(T_\infty - T_s)L^3}{\nu^2}\mathrm{Pr} = \frac{9.81\left(\frac{1}{700}\right)(1200 - 200)(1.0)^3}{(6.58 \times 10^{-5})^2}(0.697) = 2.256 \times 10^9$$

This exceeds the upper limit of the laminar quarter-power correlation in Eq. 18. Use the vertical-plate turbulent estimate for $10^9 \lesssim \mathrm{Ra}_L \lesssim 10^{13}$:

$$\overline{\mathrm{Nu}}_L = 0.10\mathrm{Ra}_L^{1/3} = 0.10(2.256 \times 10^9)^{1/3} = 131.15$$

The resulting coefficient is

$$\bar{h} = \frac{k\overline{\mathrm{Nu}}_L}{L} = \frac{0.0530(131.15)}{1.0} = 6.95\ \mathrm{W\,m^{-2}\,K^{-1}} \tag{23}$$

Churchill-Chu gives about $8.36\ \mathrm{W\,m^{-2}\,K^{-1}}$ for the same conditions. The supplied value $7.38\ \mathrm{W\,m^{-2}\,K^{-1}}$ is therefore a plausible convection estimate. The very large temperature range makes constant-property natural-convection calculations approximate. Use the supplied $h = 7.38\ \mathrm{W\,m^{-2}\,K^{-1}}$ for the requested transient calculation.

Measure $x$ from the insulated face, with the exposed face at $x = b$. The physical centre of the brick is at $x = b/2$. Reflection across the insulated face produces an equivalent symmetric slab of thickness $2b = 1.0\ \mathrm{m}$. Its half-thickness is $b$, so it is $0.50\ \mathrm{m}$, rather than $0.25\ \mathrm{m}$, that belongs in the slab Biot and Fourier numbers.

The solid diffusivity is

$$\alpha_w = \frac{k_w}{\rho_w C_{p,w}} = \frac{1.125}{2310(919)} = 5.29938 \times 10^{-7}\ \mathrm{m^2\,s^{-1}}$$

The target temperature ratio is

$$\Theta = \frac{T - T_\infty}{T_i - T_\infty} = \frac{600 - 1200}{200 - 1200} = 0.600$$

The slab Biot number is

$$\mathrm{Bi}_b = \frac{hb}{k_w} = \frac{7.38(0.50)}{1.125} = 3.28 \tag{24}$$

Internal resistance is significant, so lumped capacitance is unsuitable. For the reflected slab, the first-term plane-wall solution is

$$\Theta(x,t) = A_1 e^{-\lambda_1^2 \mathrm{Fo}} \cos\left(\frac{\lambda_1 x}{b}\right) \tag{25}$$

The eigenvalue and amplitude satisfy

$$\lambda_1 \tan\lambda_1 = \mathrm{Bi}_b \qquad A_1 = \frac{4\sin\lambda_1}{2\lambda_1 + \sin(2\lambda_1)}$$

Solving with $\mathrm{Bi}_b = 3.28$ gives $\lambda_1 \simeq 1.21582$ and $A_1 \simeq 1.21638$. At the physical centre $x/b = 0.50$, rearrange Eq. 25:

$$\mathrm{Fo} = \frac{\ln\left[\dfrac{A_1\cos\left(\frac{\lambda_1}{2}\right)}{\Theta}\right]}{\lambda_1^2} = \frac{\ln\left[\dfrac{1.21638\cos\left(\frac{1.21582}{2}\right)}{0.600}\right]}{(1.21582)^2} \simeq 0.3445 \tag{26}$$

The dimensional time follows from $\mathrm{Fo} = \alpha_w t/b^2$:

$$t = \frac{\mathrm{Fo}\, b^2}{\alpha_w} \simeq \frac{0.3445(0.50)^2}{5.29938\times10^{-7}} \simeq 1.625\times10^5\ \mathrm{s} \simeq 45.15\ \mathrm{h}$$

The calculated Fourier number exceeds 0.2, supporting the first-term approximation. The insulated face is the centre of the reflected slab, but it is not the physical centre of the original brick. At $x = 0$ the cosine is one, giving about 62.65 h to reach 600 K. These times describe different positions. If the estimated $h = 6.95\ \mathrm{W\,m^{-2}\,K^{-1}}$ replaces the stipulated value, the physical-centre result is about 46.51 h. The diffusion scale $b^2/\alpha_w \simeq 131$ h is consistent with these heating times.

Freezing $h$ is approximate because both the driving temperature difference and fluid properties change during heating. Radiation can matter much more. For example, with emissivity 0.8, a 600 K surface facing 1200 K surroundings has an effective radiation coefficient about $147\ \mathrm{W\,m^{-2}\,K^{-1}}$. The convection-only model would therefore overestimate heating time in a furnace with those radiative conditions.

**Answer**

Using the supplied constant coefficient, the physical centre of the wall reaches 600 K after approximately 45.2 h. The separate natural-convection estimate gives $h \simeq 6.95\ \mathrm{W\,m^{-2}\,K^{-1}}$, supporting the order of magnitude of the stated $7.38\ \mathrm{W\,m^{-2}\,K^{-1}}$.

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### Exercise 2. Surface temperature of a transformer tank

The tank side and top have different natural-convection characteristic lengths. The base is taken to rest on the floor, as in the lecturer's calculation.

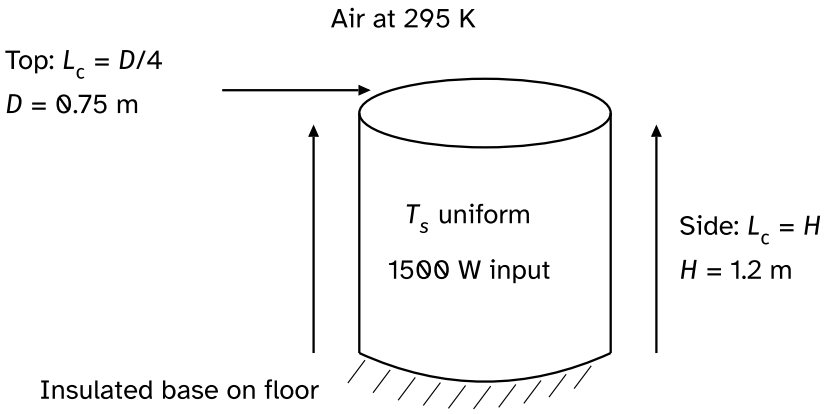



Figure 3: Cylindrical transformer tank with diameter, height, ambient temperature and separate characteristic lengths for its side and top.

**Data**

| Quantity | Value |
|---|---|
| Tank diameter | $D = 0.75\ \mathrm{m}$ |
| Tank height | $H = 1.2\ \mathrm{m}$ |
| Heat dissipation | $\dot{Q} = 1500\ \mathrm{W}$ |
| Ambient air temperature | $T_\infty = 295\ \mathrm{K}$ |
| Air conductivity at 347.5 K | $k = 0.0294\ \mathrm{W\,m^{-1}\,K^{-1}}$ |
| Air kinematic viscosity at 347.5 K | $\nu = 2.05 \times 10^{-5}\ \mathrm{m^2\,s^{-1}}$ |
| Air Prandtl number at 347.5 K | $\mathrm{Pr} = 0.718$ |
| Air conductivity at 340 K | $k = 0.0289\ \mathrm{W\,m^{-1}\,K^{-1}}$ |
| Air kinematic viscosity at 340 K | $\nu = 1.98 \times 10^{-5}\ \mathrm{m^2\,s^{-1}}$ |
| Air Prandtl number at 340 K | $\mathrm{Pr} = 0.720$ |
| Ideal-gas expansion coefficient | $\beta = 1/T_f$ |

The oil bath gives an approximately uniform surface temperature. Use the lecturer's air properties at the two trial film temperatures.

**Assumptions**

- Steady state.
- The floor-supported base is adiabatic.
- The side behaves as a vertical plate.
- Fluid properties are constant at each trial film temperature.

**Calculations**

The side area and vertical characteristic length are

$$A_{\text{side}} = \pi DH = \pi(0.75)(1.2) = 2.82743\ \text{m}^2$$

The side characteristic length is

$$L_{\text{side}} = H = 1.2\ \text{m}$$

For the upward-facing top, use area divided by perimeter:

$$A_{\text{top}} = \frac{\pi D^2}{4} = \frac{\pi(0.75)^2}{4} = 0.441786\ \text{m}^2$$

The top characteristic length is

$$L_{\text{top}} = \frac{A_{\text{top}}}{\pi D} = \frac{D}{4} = 0.1875\ \text{m}$$

The two surface heat rates add. Define their combined conductance $G$:

$$G = \bar{h}_{\text{side}} A_{\text{side}} + \bar{h}_{\text{top}} A_{\text{top}}$$

$$\dot{Q} = G(T_s - T_\infty) \tag{27}$$

The coefficients depend on the unknown surface temperature through Rayleigh number and film properties. Guess $T_s$, calculate the heat loss and adjust the guess until the heat loss equals $1500\ \text{W}$.

For the first trial, let $T_s = 400\ \text{K}$. Then $\Delta T = 105\ \text{K}$ and

$$T_f = \frac{400 + 295}{2} = 347.5\ \text{K}.$$

Using $H$ for the side gives

$$\text{Ra}_H = \frac{g\beta\Delta T H^3}{\nu^2}\text{Pr} = \frac{9.81\left(\frac{1}{347.5}\right)(105)(1.2)^3}{(2.05\times10^{-5})^2}(0.718) = 8.751\times10^9$$

Since both surfaces see the same temperature difference and fluid, the top Rayleigh number follows by scaling length cubed:

$$\text{Ra}_{D/4} = \text{Ra}_H\left(\frac{D}{4H}\right)^3 = (8.751\times10^9)\left(\frac{0.1875}{1.2}\right)^3 = 3.338\times10^7$$

For the side, use the turbulent vertical-plate correlation $\overline{\text{Nu}}_H = 0.10\text{Ra}_H^{1/3}$, applicable here above $\text{Ra}_H \simeq 10^9$. For the hot upward-facing top, use $\overline{\text{Nu}}_{D/4} = 0.15\text{Ra}_{D/4}^{1/3}$ in the range $10^7 \lesssim \text{Ra}_{D/4} \lesssim 10^{11}$.

$$
\begin{aligned}
h_{\text{side}} &= \frac{k}{H}(0.10\mathrm{Ra}_H^{1/3}) \\
&= \frac{0.0294}{1.2}[0.10(8.751\times10^{9})^{1/3}] = 5.0488\ \mathrm{W\,m^{-2}\,K^{-1}} \\
h_{\text{top}} &= \frac{4k}{D}(0.15\mathrm{Ra}_{D/4}^{1/3}) \\
&= \frac{0.0294}{0.1875}[0.15(3.338\times10^{7})^{1/3}] = 7.5732\ \mathrm{W\,m^{-2}\,K^{-1}}
\end{aligned}
\tag{28}
$$

Substitute the coefficients and areas into Eq. 27:

$$
\dot{Q}_{400} = [5.0488(2.82743) + 7.5732(0.441786)](400-295) = 1850.2\ \mathrm{W}
$$

The first guess loses more than 1500 W, so reduce the surface temperature. At $T_s = 385$ K, the film is at 340 K and $\Delta T = 90$ K. Repeating the same relations gives

$$
\begin{aligned}
\mathrm{Ra}_H &= \frac{9.81(\frac{1}{340})(90)(1.2)^3}{(1.98\times10^{-5})^2}(0.720) = 8.241\times10^{9} \\
\mathrm{Ra}_{D/4} &= (8.241\times10^{9})\left(\frac{0.1875}{1.2}\right)^3 = 3.144\times10^{7} \\
h_{\text{side}} &= \frac{0.0289}{1.2}[0.10(8.241\times10^{9})^{1/3}] = 4.8646\ \mathrm{W\,m^{-2}\,K^{-1}} \\
h_{\text{top}} &= \frac{0.0289}{0.1875}[0.15(3.144\times10^{7})^{1/3}] = 7.2968\ \mathrm{W\,m^{-2}\,K^{-1}} \\
\dot{Q}_{385} &= [4.8646(2.82743) + 7.2968(0.441786)](90) = 1528.0\ \mathrm{W}
\end{aligned}
\tag{29}
$$

For the small final correction, hold the 340 K property set fixed. Both correlations have exponent $1/3$, so $h \propto \Delta T^{1/3}$ and the heat balance gives $\dot{Q} \propto \Delta T^{4/3}$. Therefore,

$$
\Delta T = 90\left(\frac{1500}{1528.0}\right)^{3/4} = 88.760\ \mathrm{K} \tag{30}
$$

The surface temperature is

$$
T_s = 295 + 88.760 = 383.760\ \mathrm{K}.
$$

Its film temperature is 339.38 K, close to the property-table value used for the final correction. Further precision would require updating those properties. The side supplies about 81 percent of the heat loss.

Check the vertical-cylinder approximation using Eq. 20. At the corrected temperature, $\mathrm{Gr}_H \simeq 1.13\times10^{10}$, so the required diameter is

$$
D_{\min} = \frac{35H}{\mathrm{Gr}_H^{1/4}} \simeq \frac{35(1.2)}{(1.13\times10^{10})^{1/4}} \simeq 0.129\ \mathrm{m}
$$

The actual diameter $0.75\ \mathrm{m}$ exceeds this value. Radiation, excluded by the question, could be significant in an actual tank. For $\epsilon = 0.8$ and surroundings at 295 K, the radiation coefficient near the calculated temperature is about $7.2\ \mathrm{W\,m^{-2}\,K^{-1}}$, comparable to convection. Including

radiation would reduce the surface temperature. A raised tank would also lose heat from its downward-facing base, requiring that surface's own natural-convection correlation.

**Answer**

The convection-only estimate is $T_s \simeq 384$ K, or 111 °C, for an adiabatic floor-supported base.

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### Exercise 3. Cooling a horizontal metal cylinder in still air

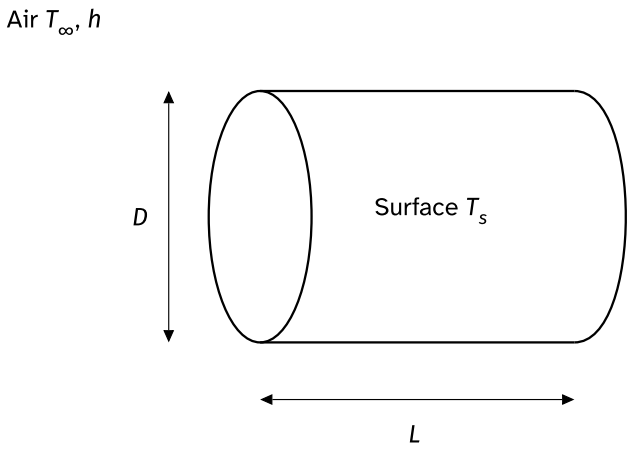



Figure 4: The horizontal cylinder loses heat from its curved surface. In the still-air case there is no imposed air speed; the fan case supplies an effective coefficient. End losses are neglected.

**Data**

| Quantity | Value |
|---|---|
| Cylinder diameter | $D = 0.180\ \mathrm{m}$ |
| Initial temperature | $T_i = 180\ ^\circ\mathrm{C}$ |
| Air temperature | $T_\infty = 30\ ^\circ\mathrm{C}$ |
| Cooling time | $t = 1200\ \mathrm{s}$ |
| Solid conductivity | $k_w = 45\ \mathrm{W\,m^{-1}\,K^{-1}}$ |
| Solid density | $\rho_w = 7800\ \mathrm{kg\,m^{-3}}$ |
| Solid specific heat capacity | $C_{p,w} = 500\ \mathrm{J\,kg^{-1}\,K^{-1}}$ |
| Air conductivity | $k = 0.032\ \mathrm{W\,m^{-1}\,K^{-1}}$ |
| Air kinematic viscosity | $\nu = 2.40 \times 10^{-5}\ \mathrm{m^2\,s^{-1}}$ |
| Air Prandtl number | $\mathrm{Pr} = 0.70$ |
| Coefficient with the fan | $h = 30\ \mathrm{W\,m^{-2}\,K^{-1}}$ |

**Assumptions**

- Uniform initial solid temperature.
- Constant properties and estimated heat transfer coefficient.
- End effects and radiation are neglected.
- Ideal-gas air for the expansion coefficient.

**Calculations**

Still air permits buoyancy-driven motion, so use natural convection around a horizontal cylinder. The initial film temperature is

$$T_f = \frac{180 + 30}{2} + 273.15 = 378.15\,\mathrm{K}.$$

For ideal-gas air, $\beta = 1/T_f = 0.002644\ \mathrm{K^{-1}}$.

The convection correlation uses the cylinder diameter:

$$\mathrm{Ra}_D = \frac{g\beta(T_i - T_\infty)D^3}{\nu^2}\mathrm{Pr} = \frac{9.81\left(\frac{1}{378.15}\right)(180-30)(0.180)^3}{(2.40\times10^{-5})^2}(0.70) = 2.758\times10^7$$

This is within the range of Eq. 21. Substitution gives

$$\overline{\mathrm{Nu}}_D = \left[0.60 + \frac{0.387(2.758\times10^7)^{1/6}}{\left[1+\left(\frac{0.559}{0.70}\right)^{9/16}\right]^{8/27}}\right]^2 = 38.17$$

The corresponding coefficient is

$$\bar{h} = \frac{k\overline{\mathrm{Nu}}_D}{D} = \frac{38.17(0.032)}{0.180} = 6.786\ \mathrm{W\,m^{-2}\,K^{-1}}$$

For the lumped-model check, use exposed volume divided by area. Ignoring ends,

$$\frac{V}{A_s} = \frac{D}{4} = 0.0450\ \mathrm{m},$$

so

$$\mathrm{Bi}_c = \frac{\bar{h}\left(\frac{V}{A_s}\right)}{k_w} = \frac{6.786(0.0450)}{45} = 0.00679 < 0.1$$

The centre is approximately at the uniform body temperature. The lumped energy balance is

$$\rho_w C_{p,w} V\frac{dT}{dt} = -\bar{h}A_s(T - T_\infty)$$

With constant $\bar{h}$, integration gives

$$T - T_\infty = (T_i - T_\infty)e^{-\frac{t}{\tau}},$$

where the time constant is

$$\tau = \frac{\rho_w C_{p,w}\left(\frac{V}{A_s}\right)}{\bar{h}} = \frac{7800(500)(0.0450)}{6.786} = 25862\,\mathrm{s}$$

At $t = 1200\,\mathrm{s}$, the temperature is

$$T(1200) = 30 + (180 - 30)e^{-\frac{1200}{25862}} = 173.2\,^{\circ}\mathrm{C}$$

Diameter in Rayleigh number and $D/4$ in the lumped Biot number answer different questions. Rayleigh number describes buoyant flow around the body; the Biot number compares internal and external thermal resistances.

With the fan, recalculate

$$\mathrm{Bi}_c = \frac{30(0.045)}{45} = 0.0300.$$

Lumped capacitance remains valid, and the same exponential gives $152.2\,^{\circ}\mathrm{C}$ after 20 minutes. If the forced coefficient had not been supplied, it would require a suitable cross-flow correlation and a new Biot check. One cannot assume that every increase in $h$ preserves lumped behaviour.

Natural-convection $h$ changes as the cylinder cools. The small 6.8 K temperature change supports this initial-coefficient estimate over 20 minutes. For a larger change, update properties and $h(T)$ in the transient balance. Radiation is excluded by the problem; this is a convection-only estimate.

**Answer**

The initial natural-convection coefficient is $\bar{h} = 6.79\,\mathrm{W\,m^{-2}\,K^{-1}}$, giving a centre temperature of approximately $173.2\,^{\circ}\mathrm{C}$ after 20 minutes. With the supplied fan coefficient, lumped capacitance remains valid and gives $152.2\,^{\circ}\mathrm{C}$.

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