CHEN3001-TP
Heat Transport - 4

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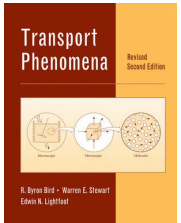
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Outline

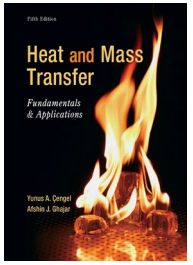
- Forced convection – Internal flow (ref-2: book chapter – 8)
  - Average velocity and temperature (ref-2: section – 8.2)
  - *h* and *Nu* definitions for internal flow in tubes and slits
  - Limiting cases
    - Constant surface heat flux (ref-2: section – 8.4)
    - Constant surface temperature (ref-2: book section – 8.4)
  - Thermal entrance region and fully developed flow (ref-2: book section – 8.3)

Ref-1: Follow the book "T. P." by Bird, Stewart, and Edwin



Ref-2: Follow the book "Heat and Mass Transfer" by Cengel and Ghajar

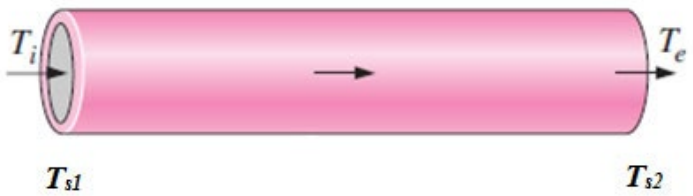
Refer to book section numbers for each topic.



Convective heat transfer – Internal flow

- h and Nu definitions for internal flow in tubes and slits

$$Q = hA_s\Delta T \xrightarrow{\text{Which } \Delta T?} Q = h_{local}A_s(T_s - T_m)$$



T_s = surface temperature
T_m = cross-sectional mean temperature

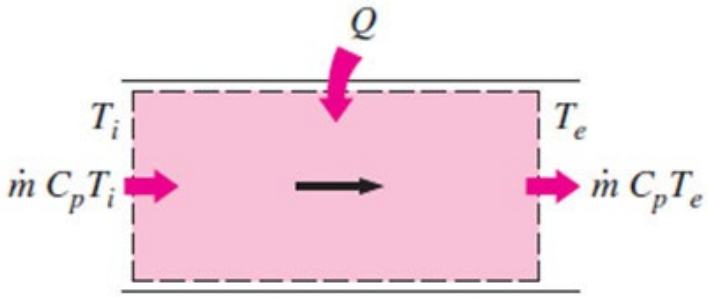
- h may also be expressed in terms of inlet and outlet temperatures:

$$Q = \overline{h}_a A_s \frac{(T_{s1} - T_i) + (T_{s2} - T_o)}{2} = \overline{h}_a A_s \Delta T_a$$ Arithmetic mean

$$Q = \overline{h}_L A_s \frac{(T_{s1} - T_i) - (T_{s2} - T_o)}{ln\left(\frac{(T_{s1} - T_i)}{(T_{s2} - T_o)}\right)} = \overline{h}_L A_s \Delta T_{ln}$$ Logarithmic mean

Convective heat transfer – Internal flow

- Internal flow – thermal analysis



Thermal energy entering and leaving C.V. (incompressible fluids & ideal gases):

$$Q = \dot{m}C_P(T_e - T_i)$$

Energy added to C.V.:

$$Q = q_s P\Delta x = h(T_s - T_m)\,P\Delta x$$

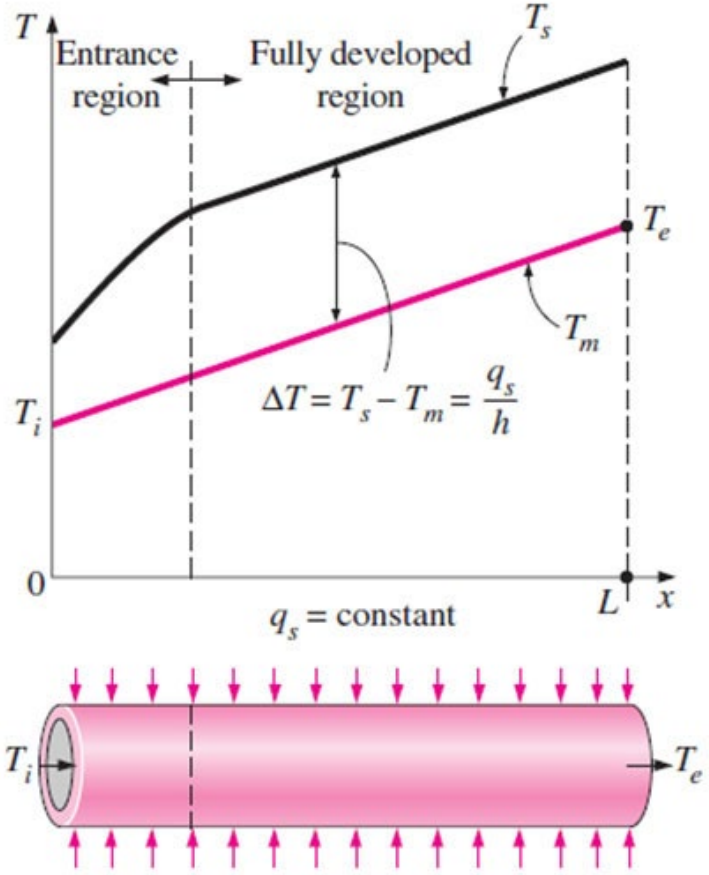
$$\dot{m}C_P(T_e - T_i) = h(T_s - T_m)\,P\Delta x \quad \Rightarrow \quad \frac{\Delta T_m}{\Delta x} = \frac{h(T_s - T_m)\,P}{\dot{m}C_P}$$

- Taking limit to $\Delta x \to 0$:

$$\frac{dT_m}{dx} = \frac{h(T_s - T_m)\,P}{\dot{m}C_P}$$

Convective heat transfer – Internal flow

- Limiting case-1 – Constant surface heat flux



Constant q_s

Examples: Tube wall is heated electrically, or irradiated at a constant heat flux

$$\frac{dT_m}{dx} = \frac{q_s\, P}{\dot{m} C_P}$$

$$T_m(x) = T_{m,i} + \frac{P}{\dot{m} C_P} q_s x$$

So, T_m varies linearly with x

What about T_s?

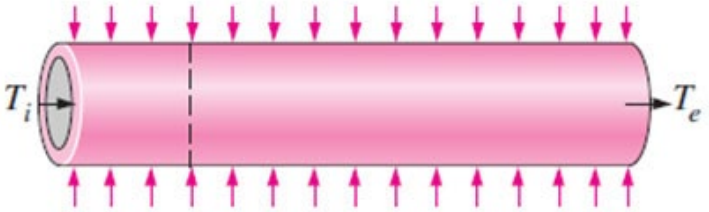
Convective heat transfer – Internal flow

- Limiting case-1 – Constant surface heat flux

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+u_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Assumptions:

- Steady state
- Laminar
- Constant *k* value
- Velocity only in the z direction
- Flow profile is already well developed at the point heat is introduced.



$$u_z\frac{\partial T}{\partial z}=\frac{k}{\rho\hat{C}_p}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

Convective heat transfer – Internal flow

- Limiting case-1 – Constant surface heat flux

$$u_z \frac{\partial T}{\partial z} = \frac{k}{\rho \hat{C}_p} \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)$$

- For steady state, laminar flow (Hagen-Poisseuille equation)

$$u_z = 2u_m\left[1 - \left(\frac{r}{R}\right)^2\right] \Rightarrow 2u_m\left[1 - \left(\frac{r}{R}\right)^2\right]\frac{\partial T}{\partial z} = \frac{k}{\rho \hat{C}_p} \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)$$

- Boundary conditions
  - $T = T_i$ at $z = 0$ for $0 \leq r \leq R$ (fluid temperature is uniform at the entrance)
  - q is constant at r = R → $\frac{\partial^2 T}{\partial z^2} = 0$
  - Symmetrical condition at the centreline → $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$

Convective heat transfer – Internal flow

- Limiting case-1 – Constant surface heat flux

$$T_m - T_s = -\frac{11}{48}\frac{q_s D}{k}$$

- With Newton's law of cooling

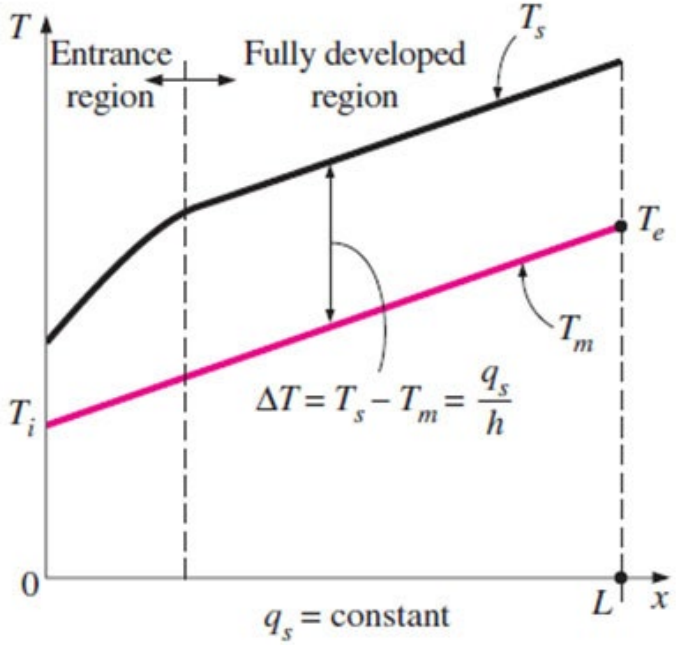
$q_s = h(T_s - T_m)$ q_s is constant in this case….

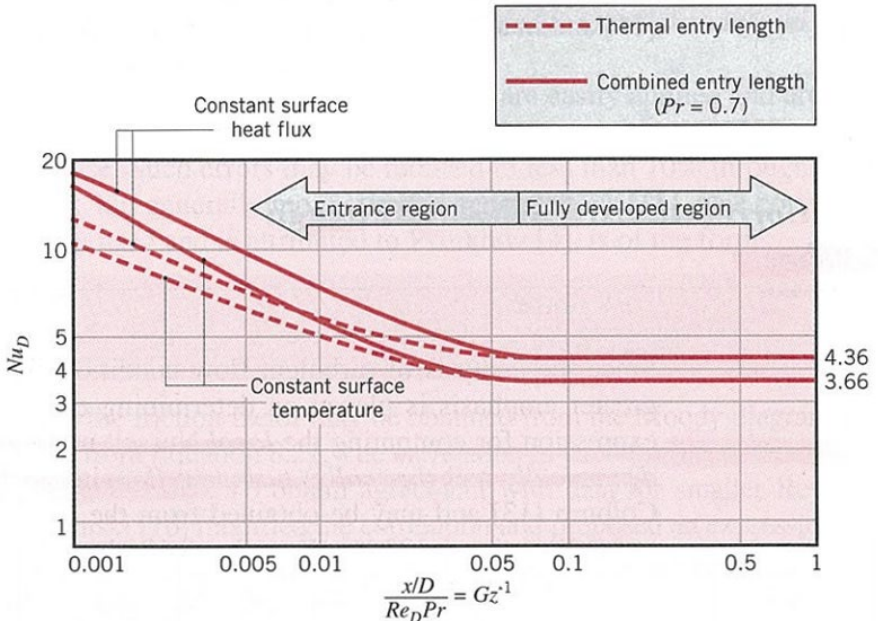
$$h = \frac{48\ k}{11\ D} \Rightarrow Nu = 4.36$$

As h and q_s are constants, $T_m - T_s$ should also be constant!

Convective heat transfer – Internal flow

- Limiting case-1 – Constant surface heat flux





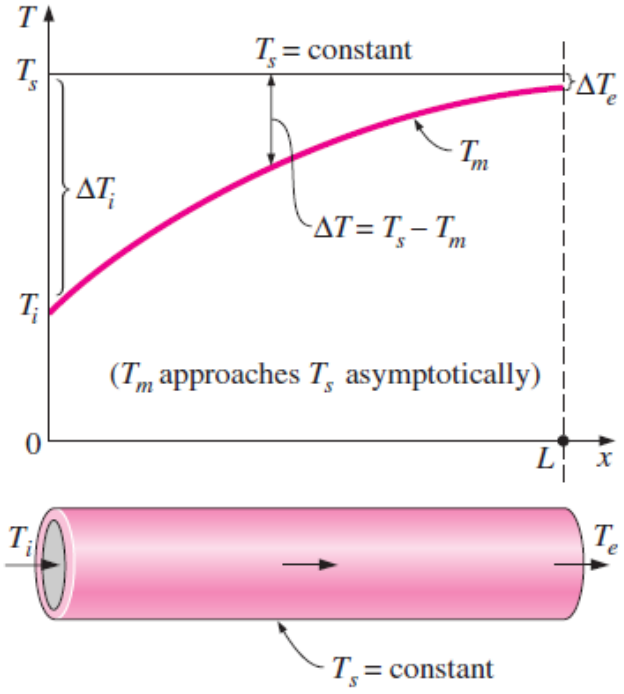
Why is the increase in T_S not linear in the entrance region?

$$q_x = h_x(T_s - T_m) \Rightarrow \text{constant}$$

Development of boundary layer:
x = 0 → B.L. thickness = 0 → very high *h* value!
As B.L. becomes well developed, *h* value decreases, and eventually becomes constant.

Convective heat transfer – Internal flow

- Limiting case-2 – Constant surface temperature



- Constant T_s
- Examples: Phase change on the outer surface of the tube (T_{sat})
- From earlier (energy balance):

$$\frac{dT_m}{dx} = \frac{h(T_s - T_m)\,P}{\dot{m}C_P}$$

$$\frac{dT_m}{dx} = -\frac{d(T_s - T_m)}{dx} = -\frac{d(\Delta T)}{dx}$$

$$\frac{d(\Delta T)}{dx} = -\frac{h(\Delta T)\,P}{\dot{m}C_P}$$

$$\frac{1}{(\Delta T)}\,d(\Delta T) = -\frac{P}{\dot{m}C_P}\,h dx$$

Convective heat transfer – Internal flow

- Limiting case-2 – Constant surface temperature

$$\frac{1}{(\Delta T)}d(\Delta T) = -\frac{P}{\dot{m}C_P}hdx \quad \Rightarrow \quad ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = -\frac{P}{\dot{m}C_P}\int_0^L hdx$$

$$ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = -\frac{P}{\dot{m}C_P}\int_0^L hdx \quad \Rightarrow \quad ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = -\frac{PL}{\dot{m}C_P}\int_0^L \frac{1}{L}hdx$$

$$ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = -\frac{PL}{\dot{m}C_P}\overline{h_L} \quad \Rightarrow \quad \frac{\Delta T_i}{\Delta T_e} = exp\left(-\frac{PL}{\dot{m}C_P}\overline{h_L}\right)$$

$$\frac{\Delta T_i}{\Delta T_e} = exp\left(-\frac{PL}{\dot{m}C_P}\overline{h_L}\right) \quad \Rightarrow \quad \frac{(T_s - T_i)}{(T_s - T_e)} = exp\left(-\frac{A_s}{\dot{m}C_P}\overline{h_L}\right)$$

Convective heat transfer – Internal flow

- Limiting case-2 – Constant surface temperature

$$ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = -\frac{A_s}{\dot{m}C_P}\overline{h_L} \Rightarrow \dot{m}C_P = -\frac{A_s}{ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}\overline{h_L}$$

$$Q = \dot{m}C_P\left((T_s - T_i) - (T_s - T_e)\right)$$

$$Q = -\frac{A_s}{ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}\overline{h_L}(\Delta T_i - \Delta T_e)$$

$$Q = A_s\overline{h_L}(\Delta T_{ln})$$

$$\Delta T_{ln} = \frac{(\Delta T_e - \Delta T_i)}{ln\left(\frac{\Delta T_e}{\Delta T_i}\right)}$$

Convective heat transfer – Internal flow

- Limiting case-2 – Constant surface temperature

$$2u_m\left[1-\left(\frac{r}{R}\right)^2\right]\frac{\partial T}{\partial z}=\frac{k}{\rho\hat{C}_p}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$ ⇨ Steady state energy balance equation

But boundary conditions are slightly different:

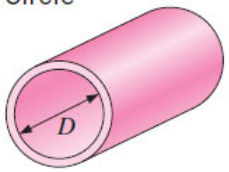
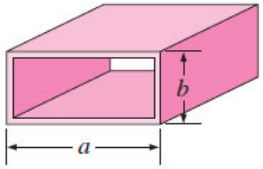
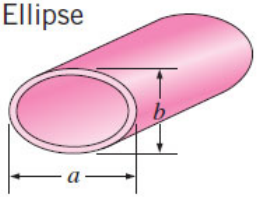
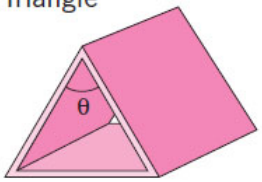
- $T = T_i$ at $x = 0$ for $0 \le r \le R$ *(Fluid temperature is uniform at the entrance)*
- $T = T_s$ at $r = R$
- Symmetrical condition at the centreline → $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$

Solution is also slightly different: $\boxed{Nu = 3.66}$

- Under laminar, steady state, and fully developed condition, Nu and h can be determined directly from equations of change
- *Nu is constant, regardless of length of pipe!*
- What about at the entrance region??

Convective heat transfer – Internal flow

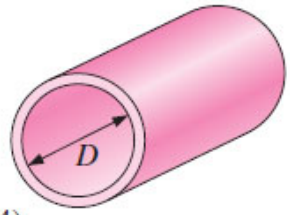
Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $\mathrm{Re} = \mathcal{V}_m D_h/\nu$, and $\mathrm{Nu} = hD_h/k$)

Tube Geometry	a/b or $\theta°$	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

Constant Nusselt number for laminar, well-developed flow!

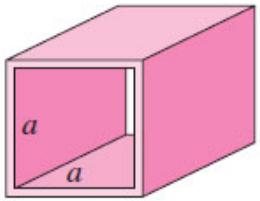
Convective heat transfer – Internal flow

Circular tube:



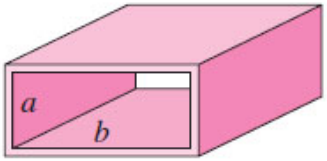
$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$

Square duct:



$$D_h = \frac{4a^2}{4a} = a$$

Rectangular duct:



$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

$$\mathrm{Re} = \frac{\rho \mathcal{V}_m D}{\mu} = \frac{\mathcal{V}_m D}{\nu}$$

For non-circular tube

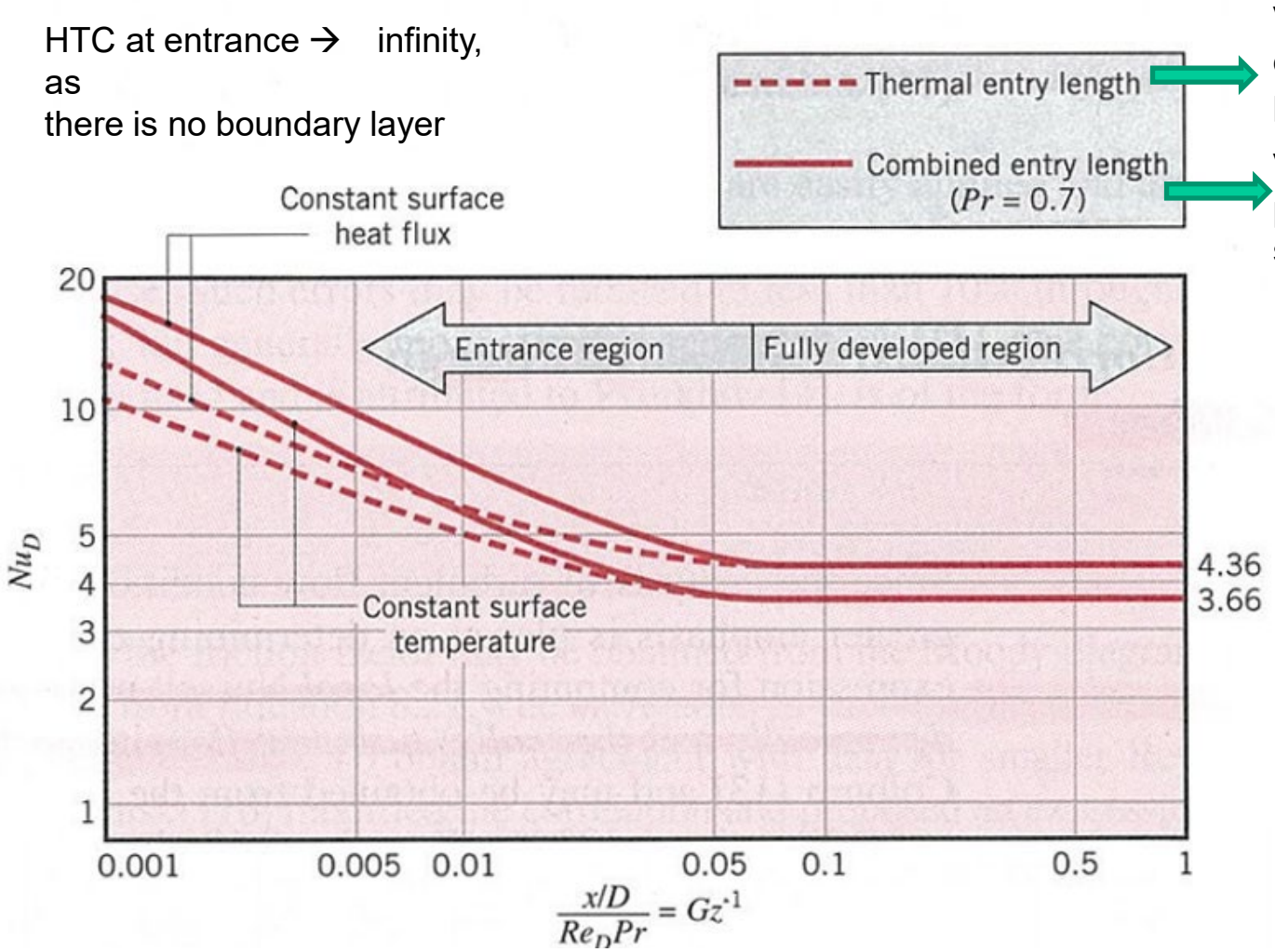
$$D_h = \frac{4A_c}{p}$$

$\mathrm{Re} < 2300$	laminar flow
$2300 \leq \mathrm{Re} \leq 10{,}000$	transitional flow
$\mathrm{Re} > 10{,}000$	turbulent flow

Convective heat transfer – Internal flow

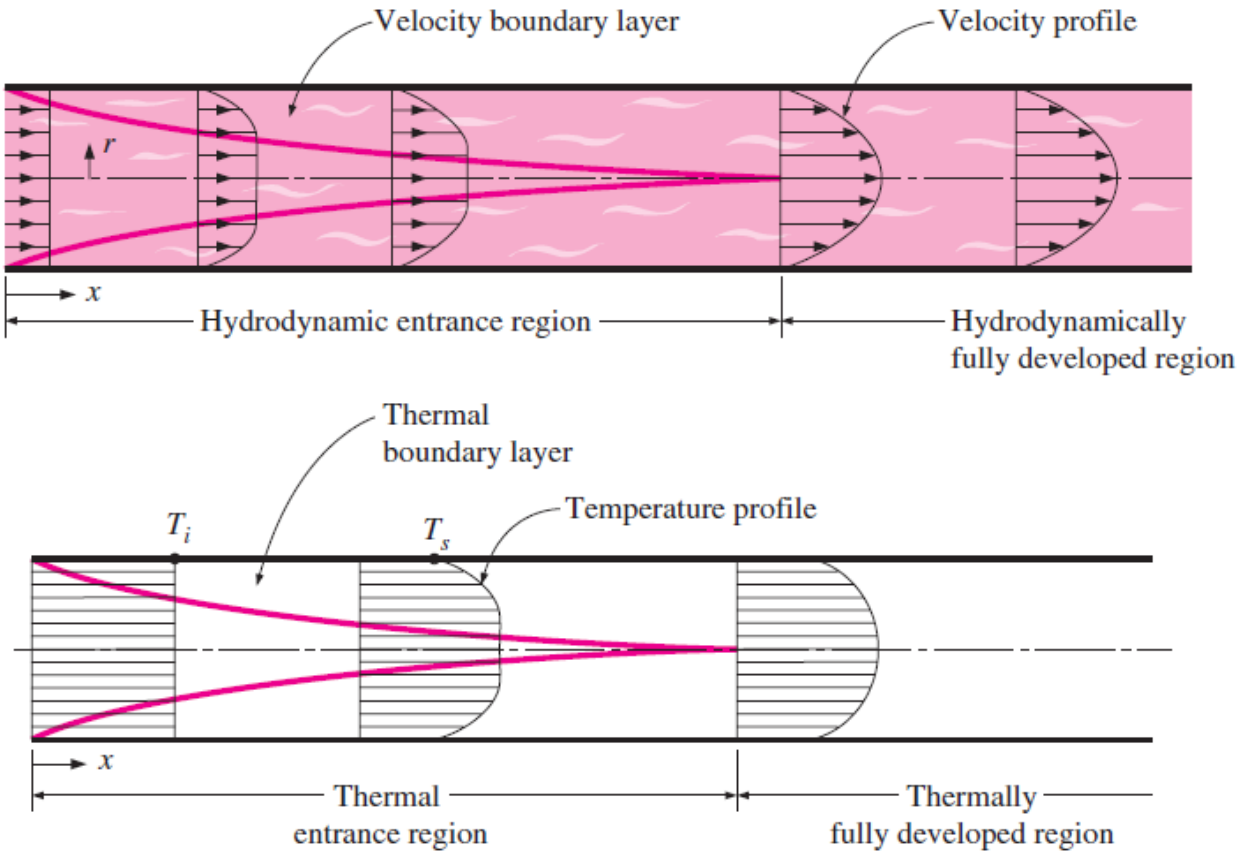
- HT Coefficient in the Entrance Region (Laminar flow)

HTC at entrance → infinity, as
there is no boundary layer



Convective heat transfer – Internal flow

- HT Coefficient in the Entrance Region (Laminar flow)



$$L_{h,laminar} \approx 0.0575 Re.D$$

$$L_{t,laminar} \approx 0.0575 Re.Pr.D$$

$$L_{h,turbulent} \approx L_{t,turbulent} \approx 50\text{D}$$

Convective heat transfer – Internal flow

- HT Coefficient in the Entrance Region
- Constant surface T & circular tube: laminar flow
  - Average Nu (Edwards et al, 1979) for thermal entry region (developed velocity profile):

$$\textit{Entry region, laminar:} \quad \mathrm{Nu} = 3.66 + \frac{0.065\,(D/L)\,\mathrm{Re\,Pr}}{1 + 0.04[(D/L)\,\mathrm{Re\,Pr}]^{2/3}}$$

  - Average Nu (Edwards et al, 1979) for combined entry region, or when the difference between the surface and the fluid T is large (large variation of viscosity with T):
    - Sieder & Tate (1936) correlation:

$$\mathrm{Nu} = 1.86\left(\frac{\mathrm{Re\,Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

All properties are evaluated at the bulk mean fluid temperature, except for μ_s, which is evaluated at the surface temperature.

Convective heat transfer – Internal flow

- HT Coefficient in the Entrance Region
- Constant surface T & circular tube: turbulent flow
  - Nu in turbulent flow?
  - Related to the friction factor, i.e. Chilton-Colburn analogy

  (Recall earlier analysis on flow over flat plate)

$$\frac{C_f}{2} = St.Pr^{2/3}$$ ⇒ Skin friction *Cf* is related to the Darcy friction factor: $$C_f = \frac{f}{4} = f_F$$

$$\frac{f_F}{2} = St.Pr^{2/3} = \left(\frac{Nu}{RePr}\right)Pr^{2/3}$$ ⇒ $$Nu = 0.125fRePr^{1/3}$$

  - For fully-developed highly turbulent flow in smooth pipe:

$$f = 0.184\ \mathrm{Re}^{-0.2}$$

$$\mathrm{Nu} = 0.023\ \mathrm{Re}^{0.8}\ \mathrm{Pr}^{1/3} \qquad \begin{pmatrix} 0.7 \le \mathrm{Pr} \le 160 \\ \mathrm{Re} > 10{,}000 \end{pmatrix}$$

Convective heat transfer – Internal flow

- HT Coefficient in the Entrance Region
- Constant surface T & circular tube: turbulent flow
  - The accuracy of Colburn equation can be improved by modifying it to Dittus-Boelter equation

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{n}$$

where $n = 0.4$ for *heating* and 0.3 for *cooling* of the fluid flowing through the tube.

  - Dittus-Boelter equation is preferred to the Colburn equation (especially for cases when the temperature difference between the fluid and the wall surface is not large)
  - When the variation in physical properties is large due to a large temperature difference, Sieder & Tate equation can be used

$$Nu = 0.027Re^{0.8}\,Pr^{\frac{1}{3}}\left(\frac{\mu}{\mu_s}\right)^{0.14} \quad \begin{pmatrix} 0.7 \le Pr \le 17600 \\ Re \ge 10000 \end{pmatrix}$$

  - The second Petukhov equation: $\mathrm{Nu} = \dfrac{(f/8)\,\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7(f/8)^{0.5}\,(\mathrm{Pr}^{2/3} - 1)} \quad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 10^4 < \mathrm{Re} < 5 \times 10^6 \end{pmatrix}$
  - At lower Re, the Gnielinski equation: $\mathrm{Nu} = \dfrac{(f/8)(\mathrm{Re} - 1000)\,\mathrm{Pr}}{1 + 12.7(f/8)^{0.5}\,(\mathrm{Pr}^{2/3} - 1} \quad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 3 \times 10^3 < \mathrm{Re} < 5 \times 10^6 \end{pmatrix}$

Convective heat transfer – Internal flow

Classroom example – 1

Air (1 atm, 77°C) enters a 5 mm I.D. tube with a bulk average velocity of 2.5 m/s. The velocity profile is developed and the thermal profile is "developing". The tube length is 1.0 m, and a constant (uniform) heat flux is imposed at the tube surface on the air over the entire length. The bulk average exit temperature of the air stream is 127 °C. Determine:

- The heat transfer coefficient at the exit (h_L)
- The heat flux imposed at the surface of the tube (q_s)
- The surface temperature (T_s) of the tube at the exit

Convective heat transfer – Internal flow

Classroom example – 2

A horizontal tubular condenser is used to condense saturated steam at 80 °C. The condenser is a shell and tube one with brass tubes (k = 110 W/m.K, 1.59 cm OD, 1.34 cm ID). Steam is outside of the tubes (h = 10760 W/m^2.K) and cooling water enters the tube at 20 °C with a velocity of 1.4 m/s. Water leaves the tubes at 40 °C. If the rate of cooling water supply is 55000 kg/h and the latent heat of condensation of steam at 80 °C is 2304 kJ/kg, calculate:

- The number of tubes required
- The length of each tube