Forced convection in internal flow

1 Errata

- Slide 12 reverses the temperature ratios in the logarithm and exponential expressions. Use the integrated balance

$$\ln\left(\frac{\Delta T_e}{\Delta T_i}\right) = -\frac{A_s \bar{h}_L}{\dot{m} C_p}$$

  Rearranging gives the temperature ratio

$$\frac{T_s - T_e}{T_s - T_i} = \exp\left(-\frac{A_s \bar{h}_L}{\dot{m} C_p}\right)$$

- Correction to the earlier worked solution for slide 22: pair the stated inlet velocity with density at the inlet, 77 degrees Celsius. Using a mean-temperature density for that velocity underestimates mass flow. The corrected wall heat flux is 159 $\mathrm{W\,m^{-2}}$ and exit wall temperature is 132.73 degrees Celsius. The PDF question itself does not print the incorrect density or answer.
- Correction to the earlier worked solution for slide 23: use cooling-water transport properties near its bulk mean of 30 degrees Celsius, with inlet density at 20 degrees Celsius for tube counting. The previous near-70-degree property set gave 79 tubes at 1.58 m. The corrected estimate is 78 tubes at 1.92 m. The PDF question itself does not print those properties or answers.

2 Reading list

- Çengel and Ghajar (2015), chapter 8.
- Bird et al. (2006).

Use q_s for inward wall heat flux, $\dot{Q}$ for heat rate, and T_i, T_e for inlet and outlet bulk temperatures.

3 Choose an internal-flow model

Given conditions	Model and checks
First establish flow and wall conditions	Use diameter in Reynolds number; distinguish prescribed wall temperature from prescribed wall flux.
Laminar and fully developed	Use Nu = 3.66 for fixed wall temperature, or Nu = 48/11 for fixed wall flux.
Laminar with thermal or combined entry	Use the length-average entrance correlation for its stated wall condition; see Eq. 20.
Turbulent flow	Check the range and mean-bulk property rule of Eq. 24 or the chosen alternative.
Outlet temperature or tube length	Use the bulk energy balance and appropriate local or average coefficient; include all resistances when using an overall coefficient.

4 Convective heat transfer: internal flow

4.1 Mean velocity, temperature and coefficient

DEFINITION
Bulk temperature

The mixing-cup temperature that gives the same enthalpy flow as the actual fluid temperature profile. For constant properties it is weighted by the local mass flow, rather than being a simple area average.

Use q for heat flux in $\mathrm{W\,m^{-2}}$ and $\dot{Q}$ for heat rate in W, as in earlier topics. The source's Q means a rate. Its T_o and T_e both mean outlet mean temperature; here use T_e, with T_i for inlet. x is axial distance in m, synonymous with the source's z, r is radial distance and $R = D/2$ the circular-tube radius. The flow area A_c and wall area A_s are in $\mathrm{m^2}$. The wetted perimeter P is in m, not pressure.

For constant density ρ in $\mathrm{kg\,m^{-3}}$, the area-mean axial velocity u_m in $\mathrm{m\,s^{-1}}$ and the mass flow $\dot{m}$ in $\mathrm{kg\,s^{-1}}$ are

$$u_m = \frac{1}{A_c}\int_{A_c} u_x \, dA$$

$$\dot{m} = \rho u_m A_c \tag{1}$$

The mixing-cup or bulk temperature T_m is defined by matching enthalpy flow. If C_p is constant, in $\mathrm{J\,kg^{-1}\,K^{-1}}$,

$$\begin{aligned} \dot{m} C_p T_m &= \int_{A_c} \rho u_x C_p T \, dA \\ T_m &= \frac{1}{u_m A_c} \int_{A_c} u_x T \, dA \end{aligned} \tag{2}$$

It is the temperature measured by catching and mixing the stream, rather than a plain area average. Fast fluid near the axis contributes more enthalpy than nearly stationary fluid near the wall. For varying properties use enthalpy itself in the definition instead of assuming $C_p T$.

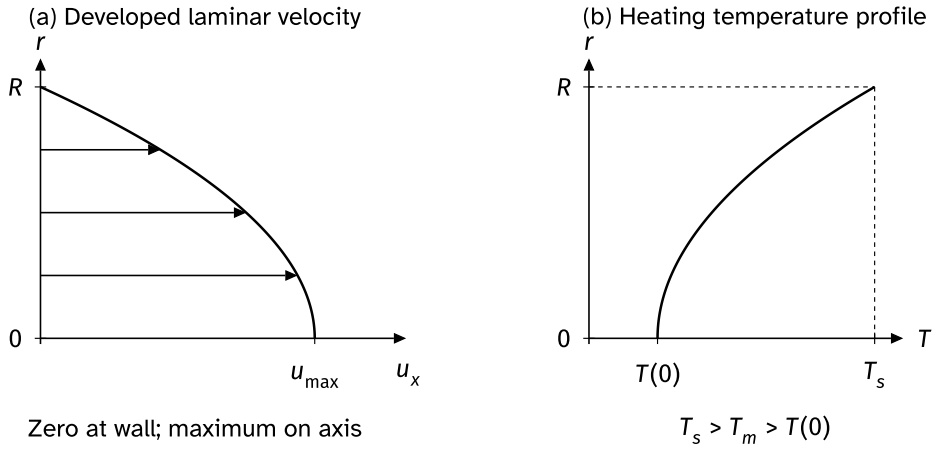


Figure 1: Developed laminar velocity and a schematic heating temperature profile from the tube axis to its wall. Radial position is vertical; velocity and temperature use separate horizontal scales. Velocity vanishes at the wall and peaks on the axis; temperature is highest at the heated wall.

Define the local coefficient by

$$q_s(x) = h(x)[T_s(x) - T_m(x)],$$

where q_s is positive heat entering the fluid. The subscript s identifies the wall; its inward sign differs from an outward radial Fourier flux. h is in $\mathrm{W\,m^{-2}\,K^{-1}}$. Conductivity k is in $\mathrm{W\,m^{-1}\,K^{-1}}$, and $\mathrm{Nu}_D = hD/k$ is dimensionless. Temperature differences in K and degrees Celsius have the same numerical value.

QUIZ

1. Bulk temperature

For constant fluid properties, what does the mixing-cup temperature preserve?

1. The temperature at the pipe axis.
2. The simple area-average temperature.
3. The wall temperature.
4. The energy carried by the flowing fluid, accounting for how velocity and temperature vary across the pipe.

See solution for Quiz 1

4.2 Thermal analysis of a pipe slice

At steady state, neglect kinetic and potential energy changes, shaft work, axial conduction and heat storage. For an incompressible liquid or ideal gas with approximately constant C_p, the whole-pipe balance is

$$\dot{Q} = \dot{m} C_p (T_e - T_i) \tag{3}$$

This conservation statement holds for either wall condition. A slice of length Δx has wall area $P\Delta x$. Equating wall input to the enthalpy rise and taking the zero-length limit gives

$$\begin{aligned} \dot{m} C_p \Delta T_m &= q_s P \Delta x \\ \dot{m} C_p \frac{dT_m}{dx} &= P q_s = P h(x) [T_s(x) - T_m(x)] \end{aligned} \tag{4}$$

More perimeter or a larger coefficient heats the stream faster. A larger capacity flow rate $\dot{m}C_p$, in $\mathrm{W\,K^{-1}}$, slows its temperature rise. Integration requires a wall condition.

QUIZ

2. Stream heating duty

A stream has capacity flow rate $\dot{m}C_p = 200\ \mathrm{W\,K^{-1}}$ and receives 1000 W without losses. What is its bulk-temperature rise?

1. 5 K.
2. 0.2 K.
3. 200 K.
4. 1000 K.

See solution for Quiz 2

4.3 Limiting case 1: constant surface heat flux

Uniform electrical heating or imposed irradiation can approximate constant q_s. With constant P and $\dot{m}C_p$, integrating Eq. 4 gives

$$T_m(x) = T_i + \frac{P q_s x}{\dot{m} C_p}$$

$$T_s(x) = T_m(x) + \frac{q_s}{h(x)} \tag{5}$$

The bulk temperature is linear even in the entrance region. The wall is linear only where h is constant. Its hottest point during heating is normally the outlet, so check outlet metal temperature for burnout or coking. A fired heater's real flux varies along and around the tube; this idealisation does not represent peak-flux maldistribution.

To calculate h, use the cylindrical energy equation from the conduction topic. Assume steady, laminar, axisymmetric flow, constant properties, an already developed axial velocity, negligible axial conduction and negligible volumetric generation or viscous dissipation. Then

$$u_x \frac{\partial T}{\partial x} = \alpha \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \tag{6}$$

The thermal diffusivity is

$$\alpha = \frac{k}{\rho C_p}$$

DEFINITION
Péclet number

The ratio of advection to diffusion over the same length L and speed U. For heat,

$$\mathrm{Pe}_L = \frac{UL}{\alpha} = \mathrm{Re}_L \mathrm{Pr}.$$

For a species,

$$\mathrm{Pe}_{m,L} = \frac{UL}{D_{AB}} = \mathrm{Re}_L \mathrm{Sc}.$$

The diffusivity must match the transported quantity. See the thermal scale estimate and the species scale estimate.

4.4 Checking neglected transport terms

For $\mathrm{Pe}_L \ll 1$, diffusion is faster than advection over the chosen length. At $\mathrm{Pe}_L \sim 1$, both terms must be retained. For $\mathrm{Pe}_L \gg 1$, advection dominates and diffusion in that direction can often be neglected away from end regions. This comparison does not remove diffusion across a tube. With $U = 1\,\mathrm{m\,s^{-1}}$, $L = 0.10\,\mathrm{m}$ and $\alpha = 2 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$, $\mathrm{Pe}_L = 5000$. If $D = 0.01\,\mathrm{m}$, the diameter-based value is only 500. Use

$$\mathrm{Pe}_L = \mathrm{Re}_D \mathrm{Pr} \frac{L}{D}$$

for the axial test, rather than substituting $\mathrm{Re}_D \mathrm{Pr}$ alone.

DEFINITION
Brinkman number

The dimensionless ratio

$$\mathrm{Br} = \frac{\mu U^2}{k \Delta T_*},$$

comparing viscous heat generation with conduction over the same transverse scale. Here μ and k are fluid properties, U is the specified velocity scale, and $\Delta T_* > 0$ is a characteristic temperature difference.

Small Br supports neglecting viscous heating; values of order one or larger require that assumption to be checked. For $\mu = 1\,\mathrm{Pa\,s}$, $U = 1\,\mathrm{m\,s^{-1}}$, $k = 0.2\,\mathrm{W\,m^{-1}\,K^{-1}}$ and $\Delta T_* =$

10 K, $\mathrm{Br} = 0.5$. That model cannot dismiss viscous heating as small. A zero wall-to-bulk temperature difference cannot be used as the denominator; select a nonzero physical scale or retain the dissipation term in the energy balance.

α is thermal diffusivity in $\mathrm{m^2\,s^{-1}}$. Steadiness removes the time derivative, axisymmetry removes angular diffusion, and developed axial flow removes radial and angular advection. Axial conduction relative to axial advection scales as

$$\frac{1}{\mathrm{Re}\,\mathrm{Pr}\left(\frac{L}{D}\right)}$$

and is below one percent when that product exceeds 100. Low Péclet numbers need another model. Viscous heating matters when the Brinkman number

$$\mathrm{Br} = \frac{\mu u_m^2}{k|\Delta T|}$$

approaches one; it can matter in polymer melts even though it is negligible in the air exercise.

The Hagen-Poiseuille profile supplied by the momentum prerequisite is

$$u_x(r) = 2u_m\left[1 - \left(\frac{r}{R}\right)^2\right] \tag{7}$$

Its area average is u_m, so centreline speed is twice the mean. The inlet condition $T(r,0) = T_i$ determines the entrance solution. In the thermally developed constant-flux region the temperature profile translates upward without changing shape. Consequently

$$\frac{\partial T}{\partial x} = \frac{dT_m}{dx}$$

is constant across the radius and

$$\frac{\partial^2 T}{\partial x^2} = 0.$$

Constant wall flux alone does not establish this in the entrance. Radial symmetry gives $\partial T/\partial r = 0$ at $r = 0$, and the inward wall flux gives

$$k\frac{\partial T}{\partial r} = q_s$$

at $r = R$.

Using $P = 2\pi R$ and $\dot{m} = \rho u_m \pi R^2$ in Eq. 4, then substituting Eq. 7 into Eq. 6, gives

$$\begin{aligned} \frac{\partial T}{\partial x} &= \frac{2q_s}{\rho C_p u_m R} \\ \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) &= \frac{4q_s}{kR}\left[1 - \left(\frac{r}{R}\right)^2\right] \end{aligned} \tag{8}$$

Multiply by r and integrate twice. A term C_1/r in the gradient would be singular on the axis, so regularity and symmetry set $C_1 = 0$:

$$\begin{aligned} r\frac{dT}{dr} &= \frac{4q_s}{kR}\left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right) \\ T &= \frac{4q_s}{kR}\left(\frac{r^2}{4} - \frac{r^4}{16R^2}\right) + C_2 \end{aligned} \tag{9}$$

Set $T(R, x) = T_s(x)$ to fix C_2. With $\eta = r/R$,

$$T - T_s = \frac{4q_s R}{k}\left(\frac{\eta^2}{4} - \frac{\eta^4}{16} - \frac{3}{16}\right) \tag{10}$$

The profile is quartic because a parabolic velocity carries energy. Apply the velocity-weighted mean Eq. 2, not an area mean:

$$\begin{aligned} T_m - T_s &= 4\int_0^1 \eta(1-\eta^2)(T - T_s)\, d\eta \\ \int_0^1 \eta(1-\eta^2)\left(\frac{\eta^2}{4} - \frac{\eta^4}{16} - \frac{3}{16}\right) d\eta &= -\frac{11}{384} \\ T_m - T_s &= -\frac{11 q_s D}{48k} \end{aligned} \tag{11}$$

For heating $T_m < T_s$, as the sign requires. Equating this difference to q_s/h from Newton's definition yields

$$h = \frac{48k}{11D}, \qquad \mathrm{Nu}_D = \frac{48}{11} = 4.3636\ldots \tag{12}$$

Nu is independent of Re and Pr under these assumptions. The dimensional h still depends on k/D. Flow rate changes the axial temperature slope rather than the developed profile's shape. Thus constant flux and constant h give parallel wall and bulk temperature lines with a fixed gap.

QUIZ

3. Constant wall heat flux

A pipe has uniform wall heat flux, constant perimeter and constant capacity flow rate. Its thermal entrance region has a changing $h(x)$. Which statement follows?

1. Both wall and bulk temperatures must be constant.
2. The bulk temperature must bend whenever the wall temperature bends.
3. Bulk temperature changes linearly along the pipe, but the difference between wall and bulk temperatures can change.
4. The wall-to-bulk gap must be zero.

See solution for Quiz 3

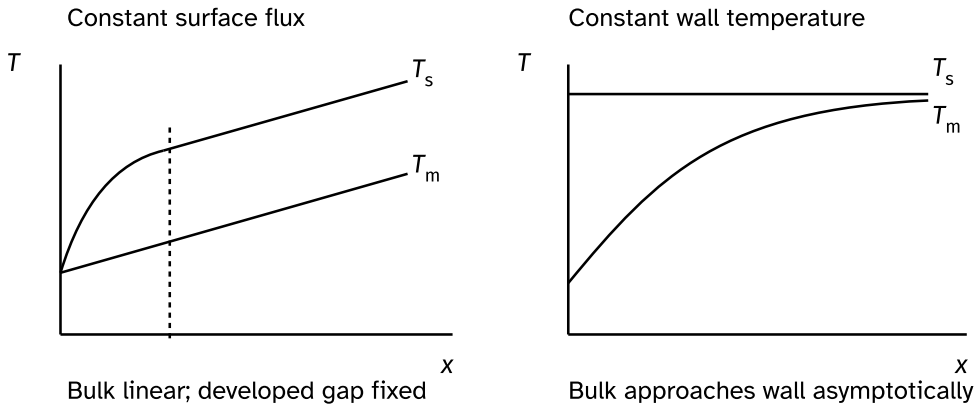


Figure 2: Comparison of wall and bulk temperatures under the two wall conditions.

Wall and bulk temperatures during heating. The entrance-region wall curve is schematic.

4.5 Limiting case 2: constant surface temperature

A pure vapour condensing or a fluid boiling at nearly fixed pressure can provide an approximately isothermal source. A much larger capacity flow rate on the other side also approximates it. Finite wall and film resistance means the tube's inner surface need not equal the source temperature, as the condenser exercise accounts for through U.

For a fixed inner-wall T_s, define $\Delta T = T_s - T_m$. Then

$$\frac{d\Delta T}{dx} = -\frac{dT_m}{dx}.$$

Substitution in Eq. 4 and separation give

$$\begin{aligned} \frac{d\Delta T}{dx} &= -\frac{Ph(x)}{\dot{m}C_p}\Delta T \\ \frac{d\Delta T}{\Delta T} &= -\frac{P}{\dot{m}C_p}h(x)\,dx \end{aligned} \tag{13}$$

DEFINITION

Number of transfer units

For heat exchange with a fixed wall temperature, the number of transfer units is

$$\mathrm{NTU}_H = \frac{\bar{h}A_s}{\dot{m}C_p}.$$

It compares surface transfer conductance with stream heat-capacity rate. For a species with fixed wall concentration on a consistent concentration basis,

$$\mathrm{NTU}_m = \frac{\bar{k}_c A_s}{Q},$$

where Q is volumetric flow rate. Both are dimensionless and describe the exponential approach to the wall state in these constant-property models.

Small NTU gives little approach to the wall state; large NTU leaves a small driving difference. NTU 0.1 leaves $e^{-0.1} = 0.905$ of the inlet difference; NTU 3 leaves $e^{-3} = 0.050$. This supports selecting the heated length. NTU alone is not the effectiveness of an arbitrary two-stream exchanger, whose capacity-rate ratio and arrangement also matter. The mass-transfer example uses the corresponding concentration difference.

The driving force decays exponentially if h is constant. Define its length-average and integrate from inlet to exit:

$$\begin{gathered} \bar{h}_L = \frac{1}{L}\int_0^L h(x)\,dx \\ \ln\frac{\Delta T_e}{\Delta T_i} = -\frac{P}{\dot{m}C_p}\int_0^L h(x)\,dx = -\frac{A_s\bar{h}_L}{\dot{m}C_p} \\ T_e = T_s - (T_s - T_i)e^{-\frac{A_s\bar{h}_L}{\dot{m}C_p}} \end{gathered} \tag{14}$$

For heating the exit difference is smaller, so the left side must be negative to match the right. The corrected outlet-over-inlet form above also handles variable h exactly through its integral. The ratio

$$\frac{A_s\bar{h}_L}{\dot{m}C_p}$$

is the number of transfer units. At a value of 3 the remaining difference is about 5 percent of its inlet value; further length gives diminishing temperature change.

Use Eq. 3 and $T_e - T_i = \Delta T_i - \Delta T_e$ to eliminate $\dot{m}C_p$ from Eq. 14:

$$\begin{gathered} \dot{Q} = A_s\bar{h}_L\Delta T_{\ln} \\ \Delta T_{\ln} = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)} \end{gathered} \tag{15}$$

For heating both differences are positive. Swapping inlet and exit leaves the mean unchanged, and equal differences have the continuous limit of their common value. For cooling, signed differences are both negative with this wall-minus-fluid convention, so $\dot{Q}$ and the signed log mean are negative; the heat-loss magnitude uses positive fluid-minus-wall differences. A crossing of zero is outside this single isothermal-wall model. A positive logarithmic mean assumes positive heating differences. An arithmetic mean overestimates the positive log mean. The latter follows from integration rather than an arbitrary choice of averaging.

QUIZ

4. Approaching an isothermal wall

A fluid enters colder than a constant-temperature wall. With positive finite h, no sources and the stated pipe balance, how does its bulk temperature behave?

1. It crosses above the wall temperature after enough length.
2. It approaches the wall temperature from below as the driving difference decreases.
3. It rises linearly without limit.
4. It immediately equals wall temperature at the inlet.

See solution for Quiz 4

4.6 Developed constant-temperature flow

DEFINITION

Graetz number

For internal thermal development,

$$\mathrm{Gz}_x = \mathrm{Re}_D \mathrm{Pr} \frac{D}{x} = \frac{UD^2}{\alpha x},$$

using mean speed U, tube diameter D and distance $x > 0$ from the start of heating. Its inverse

$$\frac{x}{D \mathrm{Re}_D \mathrm{Pr}}$$

is the dimensionless axial distance used in entrance solutions. Replace α by D_{AB} for species development.

Large Gz means little transverse diffusion time has elapsed during passage to x, so entry effects can persist. At Gz of order one, transverse diffusion has acted over a distance comparable to the diameter during that passage. Small Gz supports a developed scalar profile under the stated laminar assumptions. Use an entrance-region solution while the profile is developing, rather than a fully developed constant Nusselt number. For $\mathrm{Re}_D \mathrm{Pr} = 1000$ and $x/D = 10$, $\mathrm{Gz}_x = 100$; at $x/D = 1000$ it is one. A developed velocity profile alone does not imply a developed temperature profile. With $D = 2R$, the coordinate below is $\xi = 2/\mathrm{Gz}_x$; check factors of two before using a chart.

The same Eq. 6 and parabolic velocity now have $T(R, x) = T_s$, $T(r, 0) = T_i$ and axis symmetry. The axial gradient varies along the tube, so the constant-flux two-integration solution does not apply. The Graetz solution expands the temperature in radial eigenfunctions. To see its form, let

$$\Theta = \frac{T_s - T}{T_s - T_i}$$

and

$$\xi = \frac{\alpha x}{2u_m R^2}.$$

Substitution of

$$\Theta = F(\eta)e^{-\lambda^2 \xi}$$

gives

$$\begin{gathered}\frac{1}{\eta}\frac{d}{d\eta}\left(\eta\frac{dF}{d\eta}\right) + \lambda^2(1-\eta^2)F = 0 \\ F'(0) = 0, \qquad F(1) = 0\end{gathered} \tag{16}$$

The inlet determines expansion coefficients. Far downstream the slowest mode dominates and its first eigenvalue is $\lambda_0 \simeq 2.7044$. For that mode

$$\frac{d\Delta T_m}{dx} = -\frac{\lambda_0^2 \alpha \Delta T_m}{2u_m R^2}.$$

The slice balance independently gives

$$-\frac{2h\Delta T_m}{\rho C_p u_m R}.$$

Equating these rates gives

$$\mathrm{Nu}_D = \frac{\lambda_0^2}{2} \simeq 3.657 \simeq 3.66 \tag{17}$$

Both developed Nu values are local asymptotes, independent of pipe length. The average over an entire short tube depends on its entrance. A length ratio $L/D > 60$ can make entrance effects small for gases near Pr = 1, but is not a universal laminar guarantee for high-Pr fluids. The imposed-flux condition has a roughly 19 percent greater developed Nu than the fixed-temperature case. These are different normalized profile shapes, not a universal claim that one arbitrary heater always transfers more heat.

QUIZ

5. The wall condition in developed flow

For fully developed laminar flow in a circular tube with constant properties, which Nusselt values match the wall conditions?

1. Both wall conditions give 3.66.
2. Fixed wall temperature gives 4.36; fixed wall flux gives 3.66.
3. Either value can be used whenever the Reynolds number is below 2300.
4. Fixed wall temperature gives 3.66; fixed wall flux gives 4.36.

See solution for Quiz 5

4.7 Geometry and hydraulic diameter

DEFINITION
Hydraulic diameter

The length $D_h = 4A_c/P$ used to characterise duct flow, where A_c is the flow area and P is the wetted perimeter. It equals the diameter for a circular pipe.

A force balance gives $\Delta p A_c = \tau_w P L$, with pressure drop Δp and mean wall shear τ_w in Pa. Matching a round tube's

$$\tau_w = \frac{\Delta p D}{4L}$$

defines

$$D_h = \frac{4A_c}{P}$$

$$\mathrm{Re} = \frac{\rho u_m D_h}{\mu} = \frac{u_m D_h}{\nu} \tag{18}$$

The Prandtl number remains a fluid-property ratio:

$$\mathrm{Pr} = \frac{C_p \mu}{k} = \frac{\nu}{\alpha}$$

For a rectangle,

$$D_h = \frac{2ab}{a+b};$$

for wide parallel plates this tends to twice the gap. Hydraulic diameter does not remove laminar shape dependence. The circular-tube Nu values above apply only to circular tubes.

4.8 Example: comparing development lengths

For laminar flow, use the slide estimates

$$L_h \simeq 0.0575 \mathrm{Re} D$$

The thermal entrance-length estimate is

$$L_t \simeq 0.0575 \mathrm{Re}\,\mathrm{Pr} D \tag{19}$$

Dividing gives $L_t/L_h \simeq \mathrm{Pr}$. Oils develop thermally later than hydrodynamically, whereas gases near $\mathrm{Pr} = 1$ have comparable lengths. Check the heated length against L_t before using developed Nu.

QUIZ

6. Thermal entry length

A laminar fluid has $\mathrm{Pr} = 100$. Its velocity profile has developed, but heating has only recently begun. What should be checked before using a developed thermal Nusselt value?

1. Whether the fluid has travelled far enough from the start of heating for its temperature profile to develop.
2. Only whether the pipe has a circular cross-section.
3. Whether Prandtl number can be replaced by one.
4. Nothing, because velocity development guarantees thermal development.

See solution for Quiz 6

4.9 Laminar entrance correlations

Check Reynolds number and both entrance lengths before choosing a coefficient. The following correlations give the length-average $\bar{h}_L$ over a heated or cooled length L in a circular tube with constant wall temperature. They are not the local exit coefficient and do not apply to uniform wall flux. Use properties at the mean bulk temperature unless a wall viscosity is shown.

For developed velocity but developing temperature, slide 19 uses

$$\begin{aligned} \mathrm{Gz}_L &= \mathrm{Re}_D \mathrm{Pr} \frac{D}{L} \\ \overline{\mathrm{Nu}}_D &= 3.66 + \frac{0.065 \mathrm{Gz}_L}{1 + 0.04 \mathrm{Gz}_L^{2/3}} \end{aligned} \tag{20}$$

Use laminar flow, $\mathrm{Re}_D < 2300$, negligible axial conduction and modest property variation. The constant 3.66 supplies the developed limit as $\mathrm{Gz}_L \to 0$. The slide's rounded coefficient 0.065 is retained here and in Workshop 7; the common Hausen version uses 0.0668. These are slightly different fits, so state which one you use (Çengel and Ghajar 2015).

For appreciable viscosity variation or a combined velocity and thermal entrance, the laminar Sieder-Tate estimate is

$$\overline{\mathrm{Nu}}_D = 1.86 \mathrm{Gz}_L^{1/3} \left(\frac{\mu_b}{\mu_s} \right)^{0.14} \tag{21}$$

Use $\mathrm{Re}_D < 2300$, approximately $0.48 \leq \mathrm{Pr} \leq 16700$ and an entry-dominated length, $\mathrm{Gz}_L \gtrsim 10$. Evaluate all properties at the mean bulk temperature except μ_s, which uses the wall temperature. This is an empirical viscosity correction, not permission to ignore large changes in all properties. Do not extrapolate the entry-only expression to a long tube where it predicts a value below the developed constant-temperature limit. The laminar entrance correlations and their property rules are tabulated in Çengel and Ghajar (2015).

Flow and wall condition	Coefficient to use
Developed laminar velocity and temperature; constant wall temperature	Local developed $\mathrm{Nu}_D = 3.66$
Developed laminar velocity and temperature; constant wall flux	Local developed $\mathrm{Nu}_D = 48/11$
Developed laminar velocity; developing temperature; constant wall temperature	Length-average Eq. 20
Combined laminar entry, or a significant wall-to-bulk viscosity difference	Length-average Eq. 21 within its stated range
Turbulent flow	Select a turbulent correlation and check its range

For example, $\mathrm{Re}_D = 1000$, $\mathrm{Pr} = 7$, $D = 0.010\,\mathrm{m}$ and $L = 0.50\,\mathrm{m}$ give $\mathrm{Gz}_L = 140$ and $\overline{\mathrm{Nu}}_D = 8.04$. With $k = 0.60\,\mathrm{W\,m^{-1}\,K^{-1}}$, $\bar{h}_L = 482\,\mathrm{W\,m^{-2}\,K^{-1}}$. Using 3.66 over this whole length would miss the entrance enhancement. Put the average coefficient into the integrated isothermal-wall balance, rather than a local wall-temperature formula.

Workshop 7′s oil pipeline uses the same thermal-entry model with developed inlet velocity.

5 Turbulent flow and the friction analogy

DEFINITION
Fanning friction factor

The dimensionless factor

$$f_F = \frac{\tau_w}{\frac{1}{2}\rho u_m^2}$$

for internal flow, using mean speed u_m. It has the same wall-shear normalization as the Skin friction coefficient. The Darcy factor is $f_D = 4 f_F$.

DEFINITION
Darcy friction factor

The dimensionless pressure-loss factor in

$$\Delta p = f_D \frac{L}{D} \frac{\rho u_m^2}{2}$$

for fully developed flow in a straight constant-diameter pipe. It is four times the Fanning factor. Entrance, fitting, elevation and acceleration contributions require separate treatment.

For example, $f_F = 0.005$ means $f_D = 0.020$. Inserting 0.005 into a Darcy pressure-loss formula underpredicts that loss by four. The heat-transfer analogy below uses the same convention as its correlation.

Turbulent eddies transport momentum and energy, requiring empirical closure rather than the laminar analytical solution. The Chilton-Colburn analogy from topic 3 uses the Fanning factor $f_F = C_f$. The Stanton number is

$$\mathrm{St} = \frac{h}{\rho u_m C_p} = \frac{\mathrm{Nu}}{\mathrm{Re}\,\mathrm{Pr}}.$$

The analogy gives

$$\begin{gathered} \frac{f_F}{2} = \mathrm{St}\,\mathrm{Pr}^{2/3}, \qquad f_D = 4 f_F \\ \mathrm{Nu} = \frac{f_D}{8}\mathrm{Re}\,\mathrm{Pr}^{1/3} \end{gathered} \tag{22}$$

The distinction matters by a factor of four. Circular laminar flow has $f_D = 64/\mathrm{Re}$ and $f_F = 16/\mathrm{Re}$. Inserting the source's smooth turbulent fit $f_D = 0.184\mathrm{Re}^{-0.2}$ gives

$$\mathrm{Nu} = 0.023\mathrm{Re}^{0.8}\mathrm{Pr}^{1/3} \tag{23}$$

The coefficient is $0.184/8$ and the Re exponent is $1 - 0.2$; the Pr exponent comes from the analogy. It remains empirical because the friction fit and analogy are empirical. The source specifies $0.7 \leq \mathrm{Pr} \leq 160$, Re above 10000 and developed smooth-pipe flow, approximately $L/D > 10$. Roughness increases pressure loss without an identical heat-transfer increase, so a rough-pipe friction factor cannot be inserted without checking applicability.

5.1 Correlations used in the classroom examples

For developed, smooth-tube turbulent flow, the Dittus-Boelter estimate is

$$\begin{gathered} \mathrm{Nu} = 0.023\mathrm{Re}^{0.8}\mathrm{Pr}^{n} \\ n = 0.4 \text{ for fluid heating} \\ n = 0.3 \text{ for fluid cooling} \end{gathered} \tag{24}$$

Use bulk mean properties, $\mathrm{Re} > 10000$, $0.7 \lesssim \mathrm{Pr} \lesssim 160$, modest property variation and $L/D > 10$. The classroom examples state whether developed laminar or turbulent flow applies.

EXERCISE

Exercise 1. Electrically heated air tube

Air (1 atm, 77 °C) enters a 5 mm I.D. tube with a bulk average velocity of 2.5 m/s. The velocity profile is developed and the thermal profile is "developing". The tube length is 1.0 m, and a constant (uniform) heat flux is imposed at the tube surface on the air over the entire length. The bulk average exit temperature of the air stream is 127 °C. Determine:

- The heat transfer coefficient at the exit (h_L)
- The heat flux imposed at the surface of the tube (q_s)

- The surface temperature (T_s) of the tube at the exit

Use ideal-gas air with $R_{\text{air}} = 287.05\,\text{J/(kg K)}$ and $P = 101325\,\text{Pa}$. At the mean bulk temperature of 375.15 K, use $\mu = 2.1805 \times 10^{-5}\,\text{Pa s}$, $k = 0.03184\,\text{W/(m K)}$ and $C_p = 1011.5\,\text{J/(kg K)}$.

See solution for Exercise 1

EXERCISE
Exercise 2. Steam condenser tube sizing

A horizontal tubular condenser is used to condense saturated steam at $80\,^\circ\text{C}$. The condenser is a shell and tube one with brass tubes ($k = 110\,\text{W/(m} \cdot \text{K)}$, 1.59 cm OD, 1.34 cm ID). Steam is outside of the tubes ($h = 10760\,\text{W/(m}^2 \cdot \text{K)}$) and cooling water enters the tube at $20\,^\circ\text{C}$ with a velocity of $1.4\,\text{m/s}$. Water leaves the tubes at $40\,^\circ\text{C}$. If the rate of cooling water supply is 55000 kg/h and the latent heat of condensation of steam at $80\,^\circ\text{C}$ is 2304 kJ/kg, calculate:

- The number of tubes required
- The length of each tube

Use inlet water density $\rho_i = 998.2\,\text{kg/m}^3$ at 20 degrees Celsius. At the bulk mean of 30 degrees Celsius, use $\mu = 7.98 \times 10^{-4}\,\text{Pa s}$, $k = 0.615\,\text{W/(m K)}$ and $C_p = 4180\,\text{J/(kg K)}$. Treat 1.4 m/s as the nominal inlet speed; after rounding the number of tubes up, use the exact total mass flow to calculate the duty and Reynolds number of each tube.

See solution for Exercise 2

6 Additional examples

EXERCISE
Exercise 3. Which pipe profile develops first?

Oil enters a 16 mm diameter pipe with $\text{Re} = 900$ and $\text{Pr} = 60$. Estimate the hydrodynamic and thermal entrance lengths. Is a 5 m pipe long enough for both profiles to develop? Assume a uniform velocity and temperature at the inlet and heating beginning there.

See solution for Exercise 3

EXERCISE
Exercise 4. Compare constant wall temperature and constant wall heat flux

Water enters a $2.0\,\text{m}$ heated tube at $20\,^\circ\text{C}$. The inside diameter is $0.020\,\text{m}$, mass flow is $0.020\,\text{kg s}^{-1}$ and $C_p = 4180\,\text{J kg}^{-1}\,\text{K}^{-1}$. Adopt a constant effective coefficient $h =$

$400\ \mathrm{W\,m^{-2}\,K^{-1}}$ to isolate the effect of the wall condition. Neglect axial conduction and heat loss.

Find the outlet temperature and total heat input for two separate cases: a wall maintained at $80\ ^\circ\mathrm{C}$, and a uniform inward wall heat flux of $10000\ \mathrm{W\,m^{-2}}$. Explain how the wall-to-bulk temperature difference changes with distance in each case.

See solution for Exercise 4

7 What you should now be able to calculate

Distinguish velocity and thermal entry, calculate outlet and wall temperatures, and size tubes using consistent mass flow and an overall resistance.

Practice: Workshop 7 for water heating and developing oil flow.

8 Report an error

Email Ranjeet Utikar with the title Forced convection in internal flow, the URL `/heat-transport/internal-forced-convection/`, and the equation or passage to check.

References

Bird, R. Byron, Warren E. Stewart, and Edwin N. Lightfoot. 2006. *Transport Phenomena*. Revised 2nd. John Wiley & Sons. https://www.wiley-vch.de/en/areas-interest/engineering/transport-phenomena-978-0-470-11539-8.

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Solutions

Quiz solutions

Quiz 1. Bulk temperature

Correct choice: 4.

The bulk temperature is defined by the stream's enthalpy flow. Faster fluid carries more mass and contributes more to this mean than nearly stationary fluid near the wall. A simple area average generally gives a different value.

Back to Quiz 1

Quiz 2. Stream heating duty

Correct choice: 1.

The heat supplied raises the stream's bulk temperature:

$$T_e - T_i = \frac{\dot{Q}}{\dot{m} C_p} = \frac{1000}{200} = 5\ \mathrm{K}$$

The bulk-temperature rise is 5 K. This energy balance applies to either wall condition under the stated assumptions.

Back to Quiz 2

Quiz 3. Constant wall heat flux

Correct choice: 3.

A balance on a short pipe section gives

$$\frac{dT_m}{dx} = \frac{P q_s}{\dot{m} C_p}$$

The right-hand side is constant, so bulk temperature increases linearly with distance. The wall-to-bulk temperature difference is

$$T_s(x) - T_m(x) = \frac{q_s}{h(x)}$$

This difference changes as the heat transfer coefficient changes near the inlet.

Back to Quiz 3

Quiz 4. Approaching an isothermal wall

Correct choice: 2.

As the fluid warms, the wall-to-fluid temperature difference becomes smaller. For constant coefficient,

$$\frac{T_s - T_m(x)}{T_s - T_i} = \exp\left(-\frac{hPx}{\dot{m}C_p}\right)$$

This ratio stays positive at any finite heated length. The fluid approaches the wall temperature from below. A varying positive coefficient changes the rate of approach, but does not make the fluid hotter than the wall in this model.

Back to Quiz 4

Quiz 5. The wall condition in developed flow

Correct choice: 4.

The wall conditions produce different normalized temperature profiles. The developed circular-tube limits are 3.66 for fixed wall temperature and 4.36 for fixed wall heat flux. Entrance effects and other duct shapes need their own results.

Back to Quiz 5

Quiz 6. Thermal entry length

Correct choice: 1.

For the laminar entry-length estimates,

$$\frac{L_t}{L_h} \simeq \mathrm{Pr} = 100$$

The temperature profile can need about 100 times as much length to develop as the velocity profile. Check the distance from where heating begins. If it is too short, use a correlation that includes thermal entry effects.

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Exercise solutions

Exercise 1. Electrically heated air tube

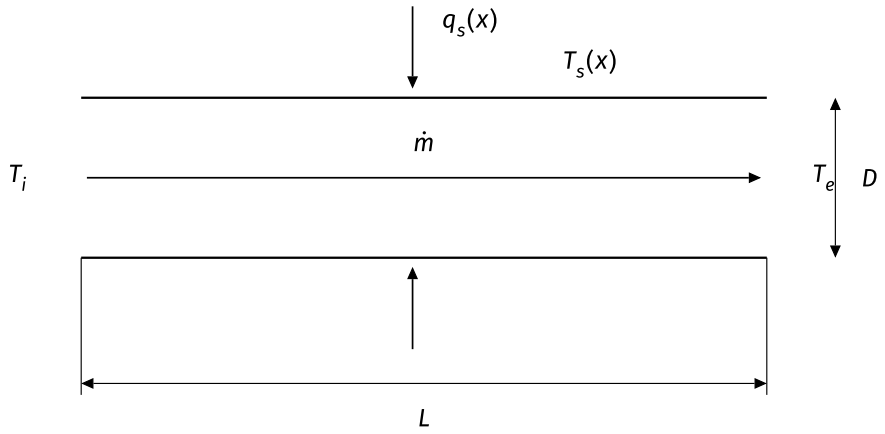


Figure 3: Heat enters through the tube wall while air flows from inlet Ti to outlet Te. In this example the inward wall flux is uniform.

Data

Quantity	Value
Inside diameter	$D = 0.0050\ \mathrm{m}$
Heated length	$L = 1.0\ \mathrm{m}$
Inlet mean speed	$u_m = 2.5\ \mathrm{m\,s^{-1}}$
Inlet bulk temperature	$T_i = 77\ ^\circ\mathrm{C}$
Outlet bulk temperature	$T_e = 127\ ^\circ\mathrm{C}$
Inlet air density	$\rho_i = 1.00810\ \mathrm{kg\,m^{-3}}$
Dynamic viscosity	$\mu = 2.1805 \times 10^{-5}\ \mathrm{Pa\,s}$
Air conductivity	$k = 0.03184\ \mathrm{W\,m^{-1}\,K^{-1}}$
Specific heat capacity	$C_p = 1011.5\ \mathrm{J\,kg^{-1}\,K^{-1}}$

The wall heat flux is uniform, and the inlet velocity profile is developed. The mean bulk temperature is 102 degrees Celsius or 375.15 K. Use transport properties near 375 K, but pair inlet velocity with inlet density:

$$\begin{aligned}\rho_i &= \frac{P}{R_{\text{air}} T_i} \\ &= \frac{101325}{287.05 \times 350.15} \\ &= 1.00810\,\text{kg}\,\text{m}^{-3}.\end{aligned}$$

Assumptions

- Steady state.
- Constant viscosity, conductivity and heat capacity; ideal-gas density varies with temperature at approximately constant pressure.
- Heat supplied at the wall enters the air.
- Axial conduction, viscous heating and kinetic and potential energy changes are neglected.
- The tube is circular and the flow remains laminar.

Calculations

First check the flow regime and whether the temperature profile has developed by the exit. Using the diameter in the Reynolds number, as defined in Eq. 18, gives

$$\text{Re}_D = \frac{\rho_i u_{m,i} D}{\mu} = \frac{1.00810(2.5)(0.0050)}{2.1805 \times 10^{-5}} = 577.91$$

The Prandtl number is

$$\text{Pr} = \frac{C_p \mu}{k} = \frac{1011.5(2.1805 \times 10^{-5})}{0.03184} = 0.69271$$

The flow is laminar. The thermal-entry estimate from Eq. 19 is

$$L_t \simeq 0.0575 \text{Re}_D \text{Pr} D = 0.0575(577.91)(0.69271)(0.0050) = 0.1151\ \text{m}$$

The temperature profile develops after heating starts, but the exit at $1.0\ \text{m}$ lies well beyond this estimated entrance length. Use the developed local constant-flux result from Eq. 12, $\text{Nu}_D = 48/11$:

$$h_L = \frac{\text{Nu}_D k}{D} = \frac{48(0.03184)}{11(0.0050)} = 27.79\ \text{W}\,\text{m}^{-2}\,\text{K}^{-1} \tag{25}$$

The total energy balance determines the wall heat flux independently of the local entrance coefficient. First calculate the air mass flow:

$$\dot{m} = \rho_i u_{m,i} \frac{\pi D^2}{4} = 1.00810(2.5) \frac{\pi (0.0050)^2}{4} = 4.94851 \times 10^{-5}\ \text{kg}\,\text{s}^{-1}$$

The enthalpy rise is the total heat input:

$$\dot{Q} = \dot{m} C_p (T_e - T_i) = (4.94851 \times 10^{-5})(1011.5)(127 - 77) = 2.5027\ \text{W}$$

Since the flux is uniform over area $\pi D L$,

$$q_s = \frac{\dot{Q}}{\pi D L} = \frac{2.5027}{\pi(0.0050)(1.0)} = 159.33\ \mathrm{W\,m^{-2}} \tag{26}$$

The local wall relation in Eq. 5,

$$q_s = h_L[T_s(L) - T_e],$$

then gives

$$T_s(L) = T_e + \frac{q_s}{h_L} = 127 + \frac{159.33}{27.79} = 132.73\ ^\circ\mathrm{C} \tag{27}$$

The wall is $5.73\ \mathrm{K}$ hotter than the exit air, as required for inward heat flow. Near the inlet the coefficient is larger, so the same heat flux needs a smaller wall-to-bulk difference. The developed coefficient must not be applied over that entrance region. The large axial Peclet product

$$\mathrm{Re}\,\mathrm{Pr}\frac{L}{D} \simeq 8.01 \times 10^4$$

also supports neglecting axial conduction. At the same volume flow and temperature rise, replacing air by water would require a much larger heat input because water has a much larger volumetric heat capacity.

Answer

At the exit, $h_L \simeq 27.8\ \mathrm{W\,m^{-2}\,K^{-1}}$. The imposed heat flux is $q_s \simeq 159\ \mathrm{W\,m^{-2}}$, and the exit wall temperature is $T_s(L) \simeq 132.7\ ^\circ\mathrm{C}$.

Back to Exercise 1

Exercise 2. Steam condenser tube sizing

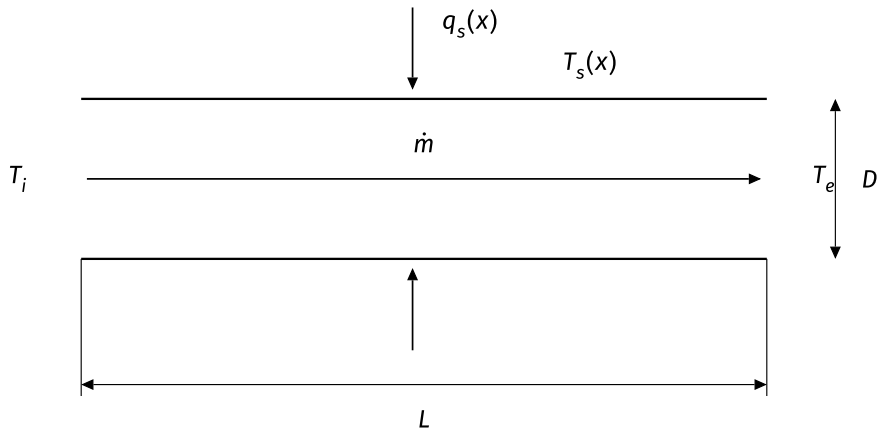


Figure 4: Water is heated as it passes through a condenser tube. The heat flux is inward; the wall temperature and local flux need not be uniform. The steam film and brass wall are included in the overall coefficient.

Data

Quantity	Value
Steam temperature	$T_{\text{steam}} = 80\,^{\circ}\text{C}$
Outer diameter	$D_o = 0.0159\,\text{m}$
Inner diameter	$D_i = 0.0134\,\text{m}$
Brass conductivity	$k_w = 110\,\text{W}\,\text{m}^{-1}\,\text{K}^{-1}$
Outside coefficient	$h_o = 10760\,\text{W}\,\text{m}^{-2}\,\text{K}^{-1}$
Water inlet temperature	$T_i = 20\,^{\circ}\text{C}$
Water outlet temperature	$T_e = 40\,^{\circ}\text{C}$
Nominal inlet water speed	$u_m = 1.4\,\text{m}\,\text{s}^{-1}$
Total water mass flow	$\dot{m} = 55000\,\text{kg}\,\text{h}^{-1}$
Inlet water density	$\rho_i = 998.2\,\text{kg}\,\text{m}^{-3}$
Dynamic viscosity	$\mu = 7.98 \times 10^{-4}\,\text{Pa}\,\text{s}$
Water conductivity	$k = 0.615\,\text{W}\,\text{m}^{-1}\,\text{K}^{-1}$
Specific heat capacity	$C_p = 4180\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$

The rounded water properties are consistent with the inlet and bulk mean states. See the IAPWS liquid-water property release. The inlet density determines flow area; mean-temperature viscosity, conductivity and heat capacity determine heat transfer.

Assumptions

- Steady state.
- Equal flow through parallel tubes.
- Constant properties and heat transfer coefficients.
- Developed turbulent flow in smooth tubes.
- Clean walls, with fouling resistance neglected.
- Negligible heat loss to the surroundings.

Calculations

The nominal inlet speed first determines a trial mass flow per tube:

$$\dot{m}_{\text{trial}} = \rho_i u_{m,i} \frac{\pi D_i^2}{4} = 998.2(1.4)\frac{\pi(0.0134)^2}{4} = 0.19708\,\text{kg}\,\text{s}^{-1}$$

Divide total flow by this value and round up:

$$N_* = \frac{\dot{m}_{\text{total}}}{\dot{m}_{\text{trial}}} = \frac{55000/3600}{0.19708} = 77.52 \tag{28}$$

Use 78 tubes. The exact mass flow per tube is then

$$\dot{m}_{\text{tube}} = \frac{55000}{3600(78)} = 0.195869\ \text{kg s}^{-1}.$$

This gives an inlet speed of $1.3914\ \text{m s}^{-1}$, close to the nominal value. Use this mass flow throughout the remaining calculation.

The inside coefficient follows from the mass flow and mean-temperature properties:

$$\text{Re}_i = \frac{4\dot{m}_{\text{tube}}}{\pi D_i \mu} = \frac{4(0.195869)}{\pi(0.0134)(7.98\times10^{-4})} = 23322$$

$$\text{Pr} = \frac{C_p\mu}{k} = \frac{4180(7.98\times10^{-4})}{0.615} = 5.4238$$

The flow is turbulent. Since the water is heated, use $n = 0.4$ in the Dittus-Boelter relation from Eq. 24:

$$\text{Nu}_i = 0.023\text{Re}_i^{0.8}\text{Pr}^{0.4} = 0.023(23322)^{0.8}(5.4238)^{0.4} = 141.14$$

Convert Nusselt number using the inside diameter:

$$h_i = \frac{\text{Nu}_i k}{D_i} = \frac{141.14(0.615)}{0.0134} = 6478\ \text{W m}^{-2}\,\text{K}^{-1} \tag{29}$$

Heat passes through the steam film, the brass wall and the water film in series. The steam temperature is fixed, but the inner wall is cooler. Using the cylindrical conduction resistance, the overall coefficient on inside area satisfies

$$\begin{aligned}\frac{1}{U_i} &= \frac{D_i}{D_o h_o} + \frac{D_i \ln\left(\frac{D_o}{D_i}\right)}{2k_w} + \frac{1}{h_i} \\ &= \frac{0.0134}{0.0159(10760)} + \frac{0.0134\ln\left(\frac{0.0159}{0.0134}\right)}{2(110)} + \frac{1}{6478} \\ &= 2.4311\times10^{-4}\ \text{m}^2\,\text{K W}^{-1}\end{aligned} \tag{30}$$

Thus $U_i = 4113\ \text{W m}^{-2}\,\text{K}^{-1}$. The water film supplies about 63.5 percent of the total resistance, the steam film 32.2 percent and the brass wall 4.3 percent. Changing the wall conductivity alone would therefore have little effect. Area bases must remain consistent, with $U_i A_i = U_o A_o$.

The driving differences are $80-20 = 60\ \text{K}$ at the water inlet and $80-40 = 40\ \text{K}$ at the outlet. Their logarithmic mean, defined in Eq. 15, is

$$\Delta T_{\ln} = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)} = \frac{60-40}{\ln\left(\frac{60}{40}\right)} = 49.326\ \text{K}$$

The per-tube heat duty follows from the water enthalpy rise:

$$\dot{Q}_{\text{tube}} = \dot{m}_{\text{tube}} C_p (T_e - T_i) = 0.195869(4180)(40-20) = 16375 \text{ W}$$

Set this equal to $U_i \pi D_i L \Delta T_{\text{ln}}$ and solve for length:

$$L = \frac{\dot{Q}_{\text{tube}}}{U_i \pi D_i \Delta T_{\text{ln}}} = \frac{16375}{4113\pi(0.0134)(49.326)} = 1.92 \text{ m} \tag{31}$$

The ratio $L/D_i \simeq 143$ supports the developed turbulent approximation. The ideal isothermal steam source needs no flow-arrangement correction to the logarithmic mean. The arithmetic mean 50 K would overestimate the driving force by about 1.37 percent.

As a steam-side check, the total water duty and the stated latent heat give

$$\dot{Q}_{\text{total}} = \frac{55000}{3600}(4180)(40-20) = 1.2772 \times 10^6 \text{ W}$$

$$\dot{m}_{\text{steam}} = \frac{\dot{Q}_{\text{total}}}{h_{fg}} = \frac{1.2772 \times 10^6}{2.304 \times 10^6} = 0.5544 \text{ kg s}^{-1} \simeq 1996 \text{ kg h}^{-1}$$

The latent heat checks the steam requirement; it does not determine tube count. These are clean-tube estimates using the stated correlation. A practical design also needs allowances for fouling, pressure drop and available tube sizes.

Answer

Using properties at the stated water temperatures, provide 78 parallel tubes of approximately 1.92 m heated length each.

Back to Exercise 2

Exercise 3. Which pipe profile develops first?

Data

- Diameter $D = 0.016$ m
- Pipe length $L = 5.0$ m
- Reynolds number $\text{Re} = 900$
- Prandtl number $\text{Pr} = 60$

Assumptions

- Uniform inlet velocity and temperature.
- Heating begins at the pipe inlet.
- Constant properties.

Calculations

The Reynolds number is below 2300, so use the laminar estimates in Eq. 19, converting the diameter to 0.016 m:

$$L_h = 0.0575(900)(0.016) = 0.828 \text{ m} \qquad L_t = 0.0575(900)(60)(0.016) = 49.7 \text{ m}$$

The velocity profile develops within the pipe, but the temperature profile does not. The ratio $L_t/L_h = \text{Pr} = 60$ expresses the slower thermal diffusion relative to momentum diffusion in

this oil. A heat transfer calculation for the 5 m pipe needs a thermal-entry correlation. A developed velocity profile alone does not justify a developed Nusselt number.

If heating starts after a long unheated approach, measure thermal entry from the start of heating. The two entrance lengths then have different origins.

Answer

The hydrodynamic entrance length is $L_h = 0.828\ \mathrm{m}$ and the thermal entrance length is $L_t = 49.7\ \mathrm{m}$. The $5.0\ \mathrm{m}$ pipe permits velocity development but remains thermally developing.

Back to Exercise 3

Exercise 4. Compare constant wall temperature and constant wall heat flux

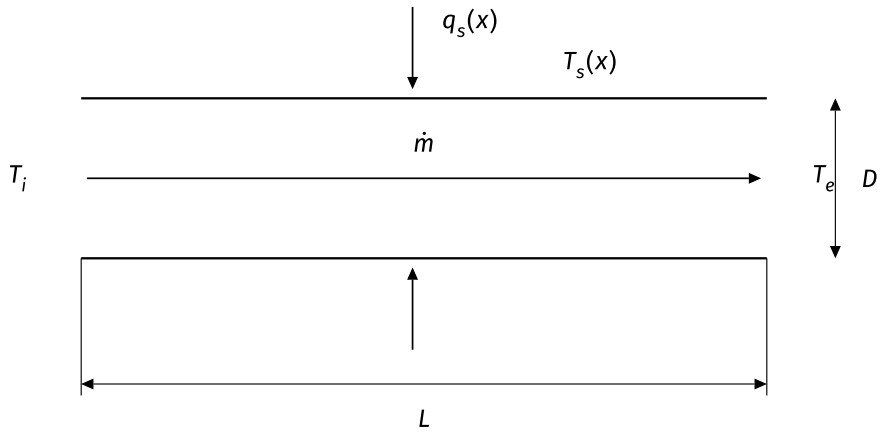


Figure 5: The same flowing stream is heated in two separate cases. One fixes the wall temperature; the other fixes inward heat flux. The energy balance relates inlet and outlet bulk temperatures in both cases.

Data

Quantity	Value
Tube length	$L = 2.0\ \mathrm{m}$
Diameter	$D = 0.020\ \mathrm{m}$
Mass flow	$\dot{m} = 0.020\ \mathrm{kg\,s^{-1}}$
Inlet temperature	$T_i = 20\ ^\circ\mathrm{C}$
Specific heat capacity	$C_p = 4180\ \mathrm{J\,kg^{-1}\,K^{-1}}$
Supplied effective coefficient	$h = 400\ \mathrm{W\,m^{-2}\,K^{-1}}$
Wall temperature in the first case	$T_s = 80\ ^\circ\mathrm{C}$
Wall heat flux in the second case	$q_s = 10000\ \mathrm{W\,m^{-2}}$

Assumptions

- Steady state.

- Constant fluid properties.
- Axial conduction and heat loss are neglected.

Calculations

The heated area is $A_s = \pi DL = 0.12566$ m² and the stream heat-capacity rate is $\dot{m}C_p = 83.6$ W K⁻¹.

For constant wall temperature, Eq. 14 gives

$$\frac{hA_s}{\dot{m}C_p} = \frac{400[\pi(0.020)(2.0)]}{0.020(4180)} = 0.6013$$

$$T_e = T_s - (T_s - T_i)e^{-\frac{hA_s}{\dot{m}C_p}} = 80 - (80 - 20)e^{-0.6013} = 47.11\,^\circ\text{C}$$

$$\dot{Q} = 83.6(47.11 - 20) = 2267\ \text{W}$$

The driving difference decreases from 60 K to 32.89 K. Even with a constant coefficient, the heat flux falls from 24000 to 13155 W m⁻². Fully developed flow does not require a constant temperature difference.

For constant wall heat flux, the overall balance instead gives

$$\dot{Q} = q_s A_s = 10000[\pi(0.020)(2.0)] = 1257\ \text{W}$$

$$T_e = T_i + \frac{\dot{Q}}{\dot{m}C_p} = 20 + \frac{1257}{83.6} = 35.03\,^\circ\text{C}$$

$$T_s - T_m = \frac{q_s}{h} = 25.0\ \text{K}$$

The bulk temperature rises linearly, and the wall temperature rises with it, from $45.0\,^\circ$C to $60.03\,^\circ$C. Their difference stays constant under the stipulated constant coefficient. In a real entrance region, $h(x)$ varies and a constant imposed flux need not give a constant difference. The two cases prescribe different heat inputs, so their duties need not agree.

Answer

For fixed wall temperature, $T_e = 47.1\,^\circ$C and $\dot{Q} = 2.27 \times 10^3$ W, with a decreasing wall-to-bulk difference. For fixed heat flux, $T_e = 35.0\,^\circ$C and $\dot{Q} = 1.26 \times 10^3$ W, with a constant 25.0 K wall-to-bulk difference.

Back to Exercise 4