# CHEN3001-TP
# Heat Transport - 3

Prepared by:
Dr Milinkumar Shah
Lecturer, Chemical Engineering Discipline
Curtin University, Western Australia

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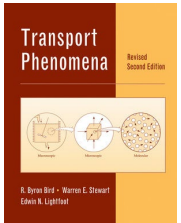
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# Outline

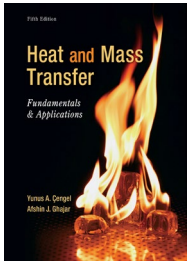
- Convective heat transfer
  - What is convective heat transfer? (ref-2: book section – 6.1)
  - Forced vs. natural convection
  - Newton's law of cooling (ref-2: book section – 6.1)
- Forced convection – External flow (ref-2: chapter – 7)
  - Determination of heat transfer coefficient
    - Empirical approach (ref-2: book section – 7.2)
    - Boundary layer approach (ref-2: book section – 7.2)
  - H.T. coefficient equations (ref-2: book section – 7.3)

Ref-1: Follow the book "T. P." by Bird, Stewart, and Edwin



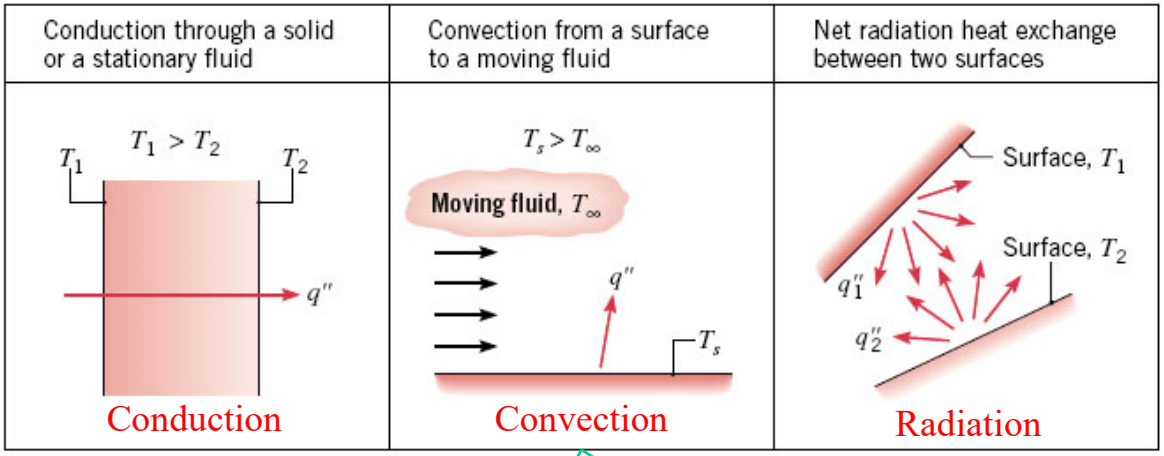

Ref-2: Follow the book "Heat and Mass Transfer" by Cengel and Ghajar

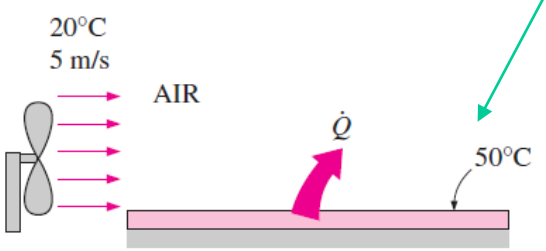
Refer to book section numbers for each topic.



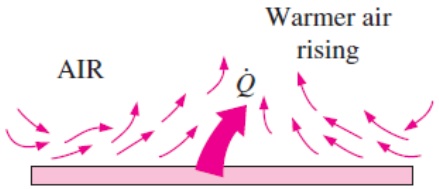
# Convective heat transfer

- What is convective heat transfer?

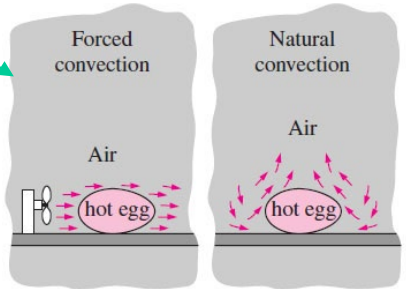


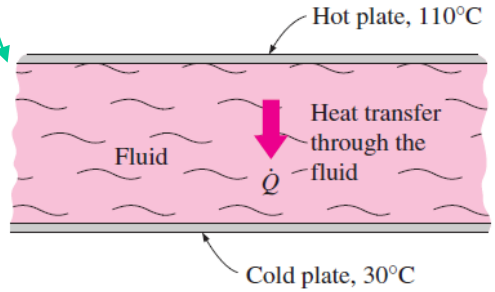




(*a*) Forced convection



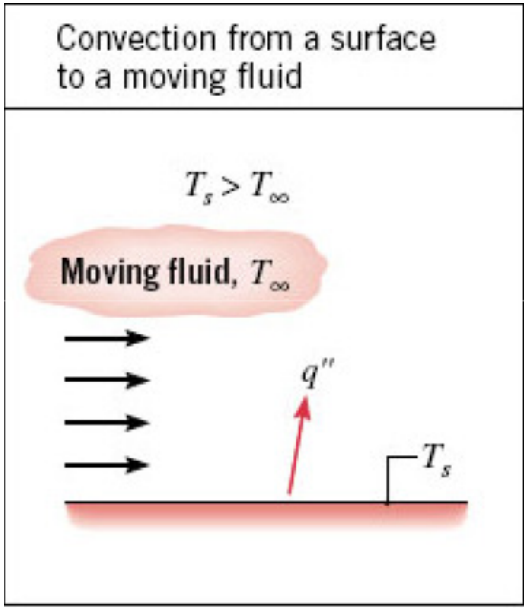

(*b*) Free convection



# Convective heat transfer

- Newton's law of cooling
  - Heat transfer between a surface & a flowing fluid




$$Q = hA(T_s - T_\infty)$$

h.t. coefficient (W/m$^2$ K)

Story so far…..

$h$ is in boundary condition in conduction problem

$$-kA\left.\frac{dT}{dx}\right|_{x=L} = hA\left(T\Big|_{x=L} - T_\infty\right)$$

$h$ is in resistances in heat exchanger

$h$ is in Biot number in unsteady conduction

  - The question is how we calculate "$h$"

# Convective heat transfer

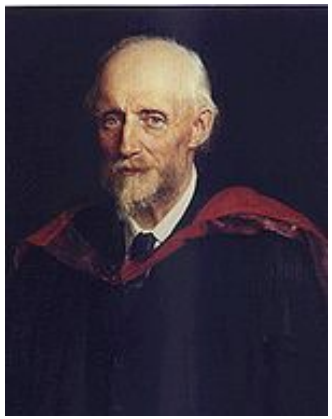
- Newton's law of cooling: heat transfer between a surface & a flowing fluid

$$Q = hA(T_s - T_\infty)$$

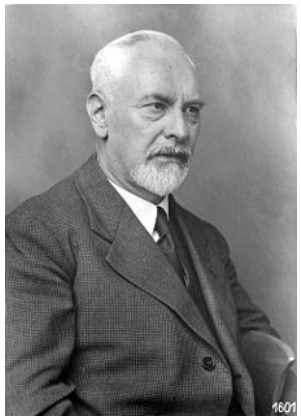
*Nu = f (Re, Pr, geometry, type of fluid flow)*



Ernst Kraft Wilhelm Nusselt
(1882-1957)



Osbourne Reynolds
(1842-1912)



Ludwig Prandtl
(1875-1953)

$$Nu = f(Re_x, Pr)$$

$$\frac{hx}{k} = f\left(\frac{\rho v x}{\mu}, \frac{c\mu}{k}\right)$$

$$Nu = \frac{hx}{k} \qquad Re_x = \frac{\rho v x}{\mu} \qquad Pr = \frac{c\mu}{k}$$

# Convective heat transfer

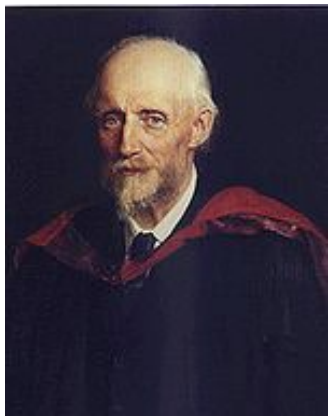
- Newton's law of cooling: heat transfer between a surface & a flowing fluid

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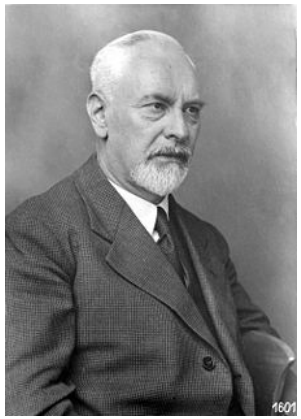
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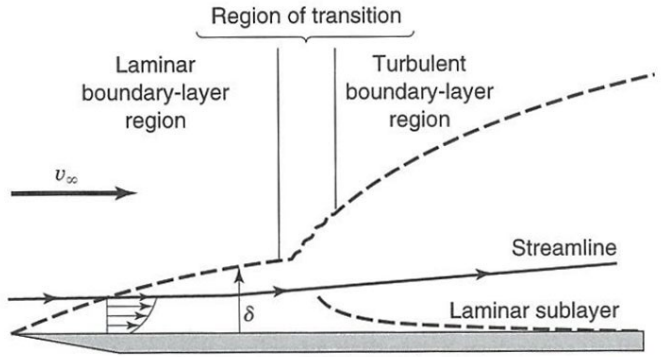


Ludwig Prandtl
(1875-1953)

$$Nu = \frac{hx}{k} \qquad Re_x = \frac{\rho v x}{\mu} \qquad Pr = \frac{c\mu}{k}$$

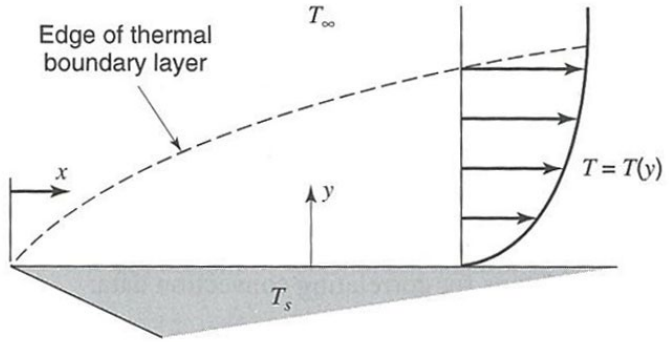
# Forced Convection: External Flow

- Boundary layer approach: consider flow over a flat plate




From fluid mech.:

- Velocity close to the plate = 0
- Distribution of velocity on the surface, until the bulk velocity is reached
- "boundary layer"

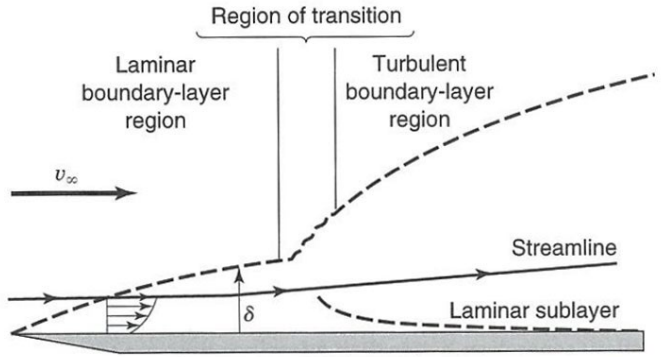


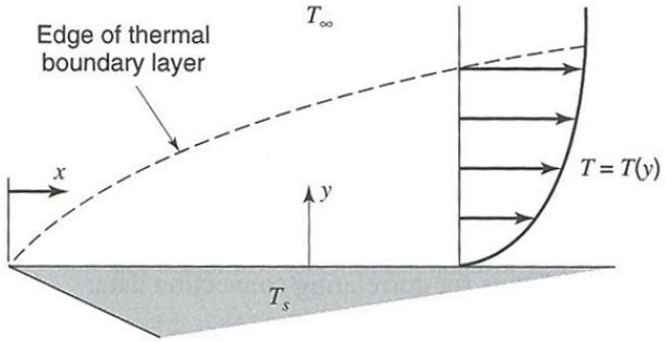

Similar phenomenon in heat transfer:

- If the temperature of the plate ≠ $T_{fluid}$
- Heat transfer due to temperature gradient
- "Thermal boundary layer"

# Forced Convection: External Flow

- Boundary layer approach: consider flow over a flat plate



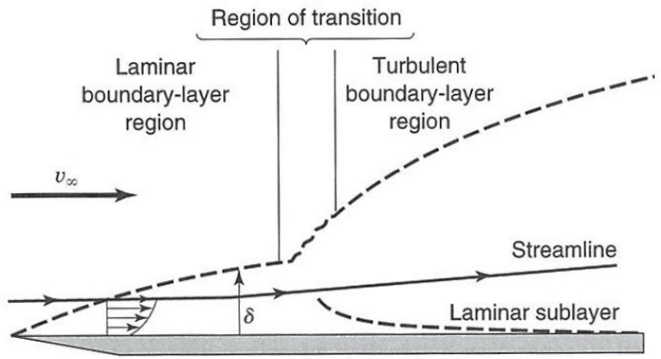




Why is "boundary layer" important?

- Highest resistance to heat transfer
- $h = f$(fluid velocity, geometry, fluid properties)
- There is a strong dependency between fluid flow and heat transfer in this region.

# Forced Convection: External Flow

- Boundary layer approach: consider flow over a flat plate

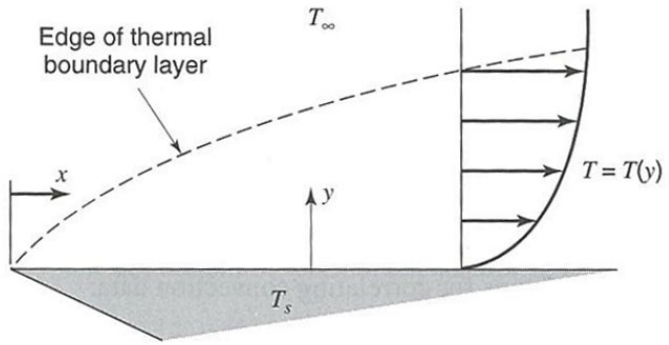



From <u>fluid mech</u>.:

- Laminar flow over a plate in isobaric condition
- Steady state condition:

$$u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = \upsilon \frac{\partial^2 u_x}{\partial y^2}$$

- Problem solved by Blasius (1908)
- Exact solution to predict the velocity profile within the boundary layer in an isothermal & laminar flow over a flat plate



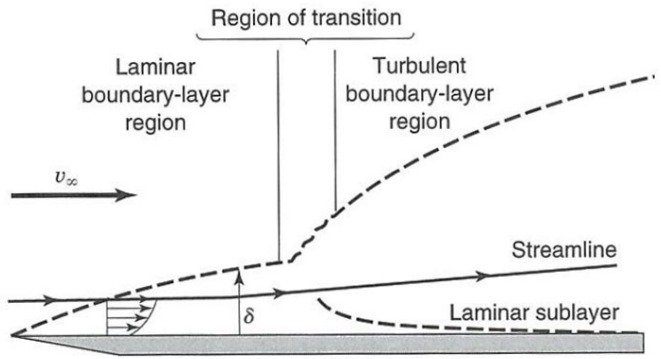

Similar phenomenon in <u>heat transfer</u>:

$$u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

If $\alpha = \upsilon$ (when $Pr = 1$), we can use Blasius' solution to analyse the temperature distribution
$\alpha$ = thermal diffusivity, $\upsilon$ = momentum diffusivity ($\mu/\rho$)

# Forced Convection: External Flow

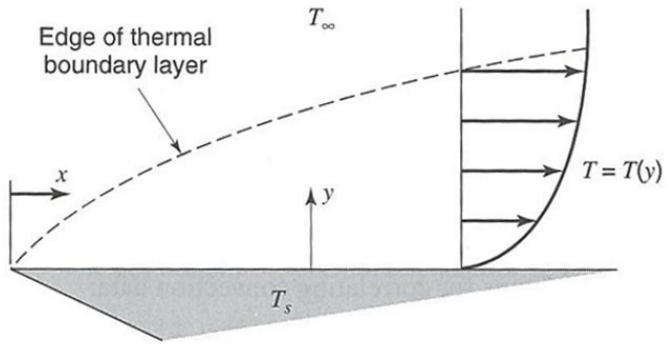
- Boundary layer approach: consider flow over a flat plate




From fluid mech.:

- Blasius' solution for velocity distribution at the surface:

$$\left.\frac{du_x}{dy}\right|_{y=0} = 0.322 u_\infty \sqrt{\frac{\rho u_\infty}{\mu x}}$$




Similar phenomenon in heat transfer:

For $Pr = 1$, the diffusion of momentum and heat are equal.

$$\left.\frac{d}{dx}\frac{u_x}{u_\infty}\right|_{y=0} = \left.\frac{d}{dx}\frac{(T - T_s)}{(T_\infty - T_s)}\right|_{y=0}$$

Therefore,

$$\left.\frac{dT}{dy}\right|_{y=0} = 0.322(T_\infty - T_s)\sqrt{\frac{\rho u_\infty}{\mu x}}$$

# Forced Convection: External Flow

- Boundary layer approach: consider flow over a flat plate

$$\left.\frac{dT}{dy}\right|_{y=0} = 0.322\frac{(T_\infty - T_s)}{x}Re_x{}^{1/2}$$

- Newton's law and Fourier's law

$$\frac{Q}{A} = h_x(T_\infty - T_s) = -k\left.\frac{dT}{dy}\right|_{y=0}$$

$$h_x = \frac{0.322k}{x}Re_x{}^{1/2}$$

Or

$$Nu = \frac{h_x x}{k} = 0.322Re_x{}^{1/2}$$ ⇒ Heat transfer coefficient in laminar flows over flat plates for *Pr = 1*

# Forced Convection: External Flow

- Boundary layer approach: consider flow over a flat plate
- For $Pr \neq 1$, Pohlhausen (1921) analysed the relation between thermal and hydrodynamic boundary layers:

$$\frac{\delta}{\delta_t} = Pr^{1/3}$$

$$\left.\frac{dT}{dy}\right|_{y=0} = 0.322 \frac{(T_\infty - T_s)}{x} Re_x{}^{1/2} Pr^{1/3}$$

$$Nu = \frac{h_x x}{k} = 0.322 Re_x{}^{1/2} Pr^{1/3}$$ ⇒ Heat transfer coefficient in laminar flows over flat plates for $Pr \neq 1$

# Forced Convection: External Flow

- Boundary layer approach: consider flow over a flat plate
- Local heat transfer coefficient & Nusselt number ($h_x, Nu_x$)
- Mean heat transfer coefficient & Nusselt number

$$Q_y = hA(T_\infty - T_s) = h_x \int (T_\infty - T_s) dA$$

$$Q_y = h(L.w)(T_\infty - T_s) = 0.322 kw Pr^{\frac{1}{3}} (T_\infty - T_s) \int_0^L \frac{Re_x^{\frac{1}{2}}}{x} dx$$

$$Nu_L = \frac{hL}{k} = 0.644 {Re_L}^{1/2} Pr^{1/3}$$

$$Nu_L = 2Nu_x$$

- Fluid properties calculated at the mean fluid temperature,

$$T_f = (T_s + T_\infty)/2$$

# Forced Convection: External Flow

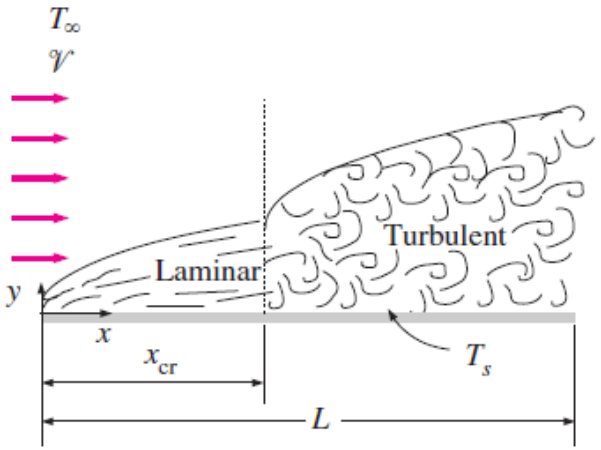
- Boundary layer approach: consider flow over a flat plate
- What happens if flow is turbulent?
  - Similar to momentum transfer: fluctuations, formation of vortices and eddies
  - Greater dispersion of heat!
  - Analysis using equation of thermal energy requires implementation of mean temperatures:

$$\overline{T} = T - T'$$

Avg. temp.

Instantaneous temp.

Fluctuating temp.




- Heat transfer not just due to molecular transfer, but also due to bulk mixing
- Higher heat transfer rate - but limited by the formation of the boundary layer

# Forced Convection: External Flow

- Boundary layer approach: consider flow over a flat plate
- Local heat transfer coefficient

$$Nu_x = \frac{h_x x}{k} = 0.322 Re_x^{0.5} Pr^{1/3} \qquad Re_x < 5 \times 10^5 \quad Laminar$$

$$Nu_x = \frac{h_x x}{k} = 0.0296 Re_x^{0.8} Pr^{1/3} \qquad 5 \times 10^5 < Re_x < 10^7 \quad Turbulent$$

(Colburn analogy)

- Average heat transfer coefficient

$$Nu_L = \frac{hL}{k} = 0.644 Re_L^{0.5} Pr^{1/3} \qquad Re_L < 5 \times 10^5 \quad Laminar$$

$$Nu_L = \frac{hL}{k} = 0.0366 Re_x^{0.8} Pr^{1/3} \qquad 5 \times 10^5 < Re_L < 10^7 \quad Turbulent$$

(Colburn analogy)

- If we have a transition from laminar to turbulent condition:

$$Nu_L = \left[0.644 Re_c^{0.5} + 0.0366\left(Re_L^{0.8} - Re_c^{0.8}\right)\right] Pr^{1/3}$$

# Forced Convection: External Flow

- Boundary layer approach: consider flow over a flat plate
  - For Pr = 1 ($C_p\mu = k$), the diffusion of momentum and heat are equal

$$\left.\frac{d}{dx}\frac{u_x}{u_\infty}\right|_{y=0} = \left.\frac{d}{dx}\frac{(T-T_s)}{(T_\infty - T_s)}\right|_{y=0} \Rightarrow C_p\mu\left.\frac{d}{dx}\frac{u_x}{u_\infty}\right|_{y=0} = k\left.\frac{d}{dx}\frac{(T-T_s)}{(T_\infty - T_s)}\right|_{y=0}$$

$$\left.\frac{C_p\mu}{u_\infty}\frac{du_x}{dx}\right|_{y=0} = \left.\frac{-k}{(T_s - T_\infty)}\frac{d(T-T_s)}{dx}\right|_{y=0}$$

Momentum flux: $\left.\mu\frac{du_x}{dx}\right|_{y=0} = C_f\frac{\rho u_\infty{}^2}{2}$

Coefficient of skin friction

$\left.-k\frac{d(T-T_s)}{dx}\right|_{y=0} = h(T_s - T_\infty)$ Heat flux

Heat transfer Coefficient

$$\left.\frac{C_p\mu}{u_\infty}\frac{du_x}{dx}\right|_{y=0} = h \Rightarrow \frac{C_p\rho C_f u_\infty}{2} = h \Rightarrow \frac{C_f}{2} = \frac{h}{\rho u_\infty C_p}$$

# Forced Convection: External Flow

- Boundary layer approach: consider flow over a flat plate

## Reynolds analogy

$$\frac{C_f}{2} = \frac{h}{\rho u_\infty C_p}$$

Stanton number, St

Conditions of validity:

- Pr = 1
- No form drag (i.e. drag caused by pressure gradients)

Applicable to laminar and turbulent flow conditions

## Colburn analogy

$$\frac{C_f}{2} = St.Pr^{2/3}$$

Conditions of validity: Pr ≠ 1

*In relation to Pohlhausen (1921) theory:*

$$\frac{C_f}{2} = \left(\frac{Nu}{Re.Pr}\right)Pr^{2/3} = \left(\frac{Nu}{Re.Pr^{1/3}}\right)$$

$$Nu = \frac{C_f}{2}Re.Pr^{1/3}$$

# Forced Convection: External Flow

- Boundary layer approach: consider flow over a flat plate

Classroom examples-1

Air at a pressure of 6 kN/m$^2$ and temperature of 300 °C flows with a velocity of 10 m/s over a flat plate 0.5 m long. Estimate the cooling rate per unit width of the plate needed to maintain the surface at a temperature of 27 ° C.

- $v_{@1\,atm} = 30.84 \times 10^{-6}$ m$^2$/s
- $k_{air\ at\ 165\,℃} = 3.6427 \times 10^{-2}$ W/m.K
- $Pr_{air\ at\ 165\,℃} = \sim 0.684$

# Forced Convection: External Flow

- Boundary layer approach: consider flow over a sphere
- Consider a heated sphere of radius R is suspended in a large, motionless body of fluid.
- Set up a differential equation describing the temperature in the surrounding fluid as a function of *r*, the distance from the centre of the sphere.
- B. C.'s
  - At r = R, $T = T_R$
  - At r = ∞, $T = T_\infty$

- Consider that the thermal conductivity of the fluid is constant.

$$\rho C_p \frac{DT}{Dt} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \emptyset}\left(k\frac{\partial T}{\partial \emptyset}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right) + S$$

# Forced Convection: External Flow

- Boundary layer approach: consider flow over a sphere
- Consider a heated sphere of radius R is suspended in a large, motionless body of fluid.
- Heat conduction in the r-direction:

$$\rho C_p \frac{DT}{Dt} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \emptyset}\left(k\frac{\partial T}{\partial \emptyset}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right) + S$$

$$\frac{k}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = 0 \Rightarrow \frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = 0$$

(As $k$ is constant)

- Integrate: $r^2\frac{\partial T}{\partial r} = 0 \Rightarrow T = -\frac{C_1}{r} + C_2$
- With B.C. 2: At r = ∞, T = $T_\infty$ ⇒ $C_2 = T_\infty$
- With B.C. 1: At r = R, T = $T_R$ ⇒ $C_1 = R(T_\infty - T_R)$

$$\frac{T - T_\infty}{T_R - T_\infty} = \frac{R}{r}$$

# Forced Convection: External Flow

- Boundary layer approach: consider flow over a sphere
- Consider a heated sphere of radius R is suspended in a large, motionless body of fluid.
- Heat flux at the surface:

$$q_r\Big|_{r=R} = -k\frac{dT}{dr}\Big|_{r=R} = -k\frac{d}{dr}\left(\frac{R}{r}(T_R - T_\infty) + T_\infty\right)\Big|_{r=R}$$

$$= kR(T_R - T_\infty)\frac{1}{r^2}\Big|_{r=R}$$

$$= \frac{k(T_R - T_\infty)}{R}$$

- Equating to convective heat flux:

$$q_r\Big|_{r=R} = \frac{k(T_R - T_\infty)}{R} = h(T_R - T_\infty) \Rightarrow h = \frac{k}{R} \Rightarrow Nu = \frac{hD}{k} = 2$$

# Forced Convection: External Flow

- Boundary layer approach: consider flow over a sphere
- At higher Reynolds number:
- Ranz and Marshall (1952)

$$Nu = 2 + 0.6Re^{0.5}Pr^{1/3}$$

Generally used for predicting heat transfer to and from droplets and bubbles

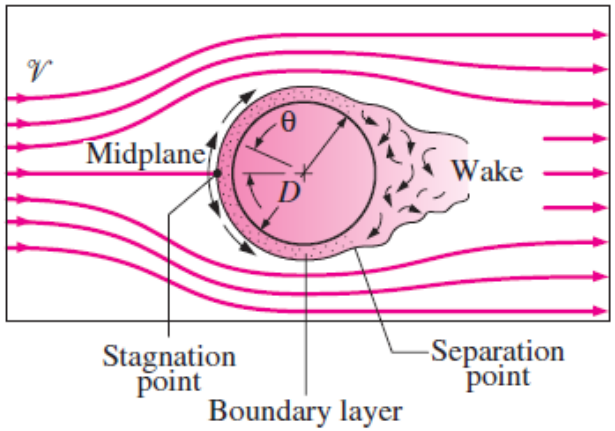
- Whitaker (1977)

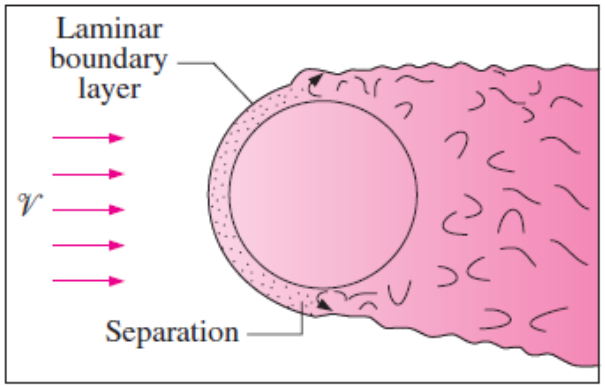
$$Nu = 2 + \left(0.4Re^{0.5} + 0.06Re^{2/3}\right)Pr^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

All physical properties are evaluated at the *approaching stream temperature*, except for μS, which is evaluated based on the flow conditions at the surface of the sphere.

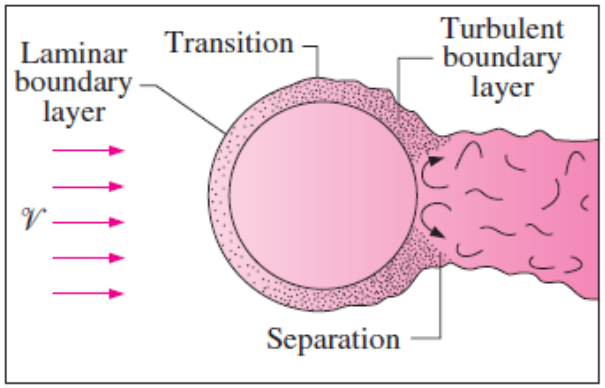
# Forced Convection: External Flow

- Boundary layer approach: consider flow across a cylinder

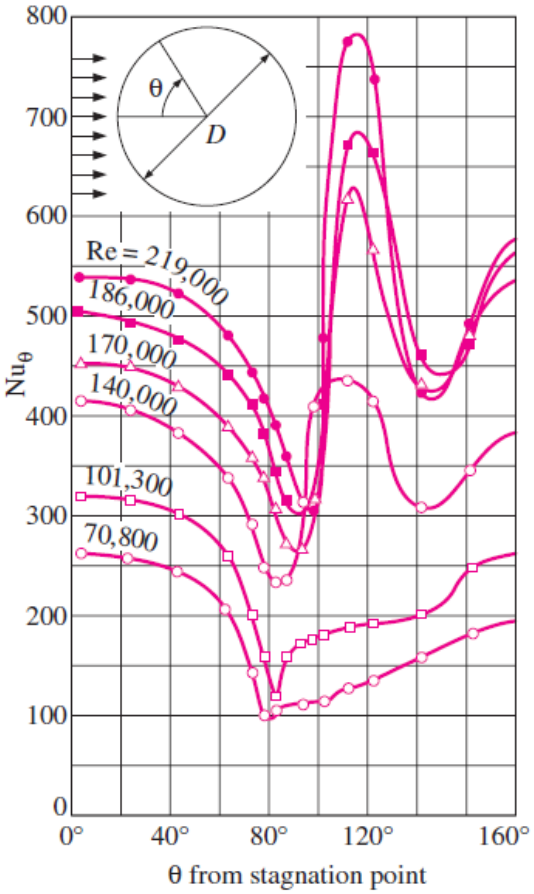



(*a*) Laminar flow ($Re < 2 \times 10^5$)




(*b*) Turbulence occurs ($Re > 2 \times 10^5$)




- Local *h*
- Stagnation point at the front of the cylinder
- Decrease in *h* as $\theta > 0$ as the laminar boundary layer is thicker
- With turbulent flows: flow separation causes increase in *h*

# Forced Convection: External Flow

- Boundary layer approach: consider flow across a cylinder

- Average h (Churchill and Bernstein)

$$Nu_{cyl.} = \frac{hD}{k} = 0.3 + \frac{0.62Re^{1/2}Pr^{1/3}}{[1+(0.4/Pr)^{2.3}]^{1/4}}\left[1+\left(\frac{Re}{282000}\right)^{5/8}\right]^{4/5}$$

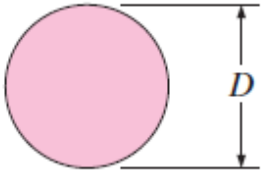
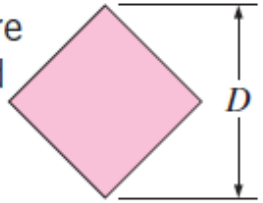
- Alternatively, we can use

$$Nu_{cyl.} = \frac{hD}{k} = CRe^{m}Pr^{n}$$

- $n$ = 0.33, $C$ and $m$ are determined experimentally based on Re value.

# Forced Convection: External Flow

- Boundary layer approach: consider flow across a cylinder

| Cross-section of the cylinder | Fluid | Range of Re | Nusselt number |
|---|---|---|---|
| Circle   | Gas or liquid | 0.4–4<br>4–40<br>40–4000<br>4000–40,000<br>40,000–400,000 | $Nu = 0.989Re^{0.330}\ Pr^{1/3}$<br>$Nu = 0.911Re^{0.385}\ Pr^{1/3}$<br>$Nu = 0.683Re^{0.466}\ Pr^{1/3}$<br>$Nu = 0.193Re^{0.618}\ Pr^{1/3}$<br>$Nu = 0.027Re^{0.805}\ Pr^{1/3}$ |
| Square   | Gas | 5000–100,000 | $Nu = 0.102Re^{0.675}\ Pr^{1/3}$ |
| Square (tilted 45°)   | Gas | 5000–100,000 | $Nu = 0.246Re^{0.588}\ Pr^{1/3}$ |

# Forced Convection: External Flow

- Classroom example-2
- A metallic cylinder (12.7 mm in diameter and 94 mm long) is heated internally by an electrical heater and is subjected to a cross flow of air. The air velocity and temperature are maintained at 10 m/s and 26.2 °C. The heater power dissipation is measured to be 39.1 W, while the average cylinder surface temperature is 128.4 °C. Compare the heat transfer coefficient from experimental observations and correlation predictions.

