Forced convection in external flow

1 Errata

- Slides 11, 12, 13 and 16 give 0.322 as the local laminar coefficient. Use 0.332.

  Slides 14 and 16 give 0.644 as the average laminar coefficient. Use 0.664.

- Slides 11 and 17 use d/dx for the normalized velocity and temperature gradients. Use d/dy.

  These gradients are normal to the wall.

- Slide 14 takes the varying coefficient h_x outside the heat-rate integral. Keep it inside the integral and use

$$Q = \int_A h_x (T_\infty - T_s)\, dA$$

- Slide 16 uses Re_x in the turbulent average correlation. Use Re_L.

  The average correlation uses the full plate length.

- Slides 20 and 21 omit $\sin\theta$ inside the derivative in the polar diffusion term. Use

$$\frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(k \sin\theta \frac{\partial T}{\partial \theta} \right)$$

- Slide 21 sets the result of the first integration to zero. Use

$$r^2 \frac{dT}{dr} = C_1$$

  The integration introduces a constant whose value depends on the boundary conditions.

- Slide 23 gives 1977 as the year of the Whitaker correlation. Use 1972.
- Slide 25 gives 2.3 as the Churchill-Bernstein denominator exponent. Use $2/3$.

2 Reading list

- Çengel and Ghajar (2015), chapters 6 and 7.
- Bird et al. (2006).

Here k is fluid conductivity, q is heat flux and $\dot{Q}$ is heat rate.

3 Choose an external-flow correlation

Given conditions	Model and checks
Local heat flux on a flat plate	Use the local coefficient at x, Eq. 5; base Reynolds number on x.
Heat loss from a whole flat plate	Use a length-average coefficient, Eq. 7 or Eq. 10; base Reynolds number on L.
Forced flow past an isolated sphere	Use Eq. 19 or Eq. 20; check diameter, relative speed, range and property temperature.
Cross-flow over a cylinder	Use a cylinder correlation and diameter; do not substitute the sphere relation.

4 Deriving the flat-plate coefficient

Newton's coefficient is defined by

$$q = h(T_s - T_\infty) \tag{1}$$

The total heat rate over an isothermal surface is

$$\dot{Q} = \bar{h} A (T_s - T_\infty)$$

DEFINITION
Forced convection

Convection in which a pump, fan or imposed stream drives the fluid motion.

DEFINITION
Reynolds number

The dimensionless ratio

$$\mathrm{Re} = \frac{\rho u L}{\mu} = \frac{u L}{\nu},$$

comparing inertial and viscous effects. The velocity and characteristic length must suit the flow geometry.

For $\mathrm{Re} \ll 1$, viscous effects dominate and a creeping-flow model can neglect inertia. At large Re, inertia is more important and thin viscous boundary layers can form near surfaces. Large Re alone does not identify a flow regime without its geometry. For this plate use free-stream speed and distance from the leading edge. With $u_\infty = 2\,\mathrm{m\,s^{-1}}$, $x = 0.30\,\mathrm{m}$ and $\nu = 1.5 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$, $\mathrm{Re}_x = 40000$. This is below the nominal plate transition estimate used below. Substituting the pipe threshold of 2300 would incorrectly classify this plate flow.

DEFINITION
Nusselt number

The dimensionless heat transfer coefficient $\mathrm{Nu} = hL/k_f$. It compares surface heat transfer with conduction across a fluid length L and uses the fluid conductivity k_f.

Nu measures the wall temperature gradient relative to the reference gradient across L. A larger value means stronger transfer on that scale; a value of order one means the reference conduction scale is comparable. There is no universal stagnant value of one. The stagnant-sphere calculation below gives two when L is diameter. For $h = 20\ \mathrm{W\,m^{-2}\,K^{-1}}$, $L = 0.10\ \mathrm{m}$ and $k_f = 0.025\ \mathrm{W\,m^{-1}\,K^{-1}}$, $\mathrm{Nu} = 80$. Using plate conductivity here would describe the wrong resistance. Use h_x with x for a local value, or $\bar{h}_L$ with full length L for an average value. The solid resistance is compared by Biot number in transient conduction.

DEFINITION
Prandtl number

The dimensionless ratio $\mathrm{Pr} = \nu/\alpha$, comparing momentum diffusivity with thermal diffusivity.

For $\mathrm{Pr} \ll 1$, heat diffuses faster than momentum, so the thermal boundary layer extends beyond the velocity boundary layer. For $\mathrm{Pr} \gg 1$, heat diffuses more slowly and the thermal layer is thinner. At $\mathrm{Pr} \approx 1$, the two diffusion rates and boundary-layer thicknesses are comparable under the plate assumptions below. For $\nu = 10^{-6}$ and $\alpha = 10^{-7}\ \mathrm{m^2\,s^{-1}}$, $\mathrm{Pr} = 10$, so the thermal layer is thinner in the stated plate model. Pr is a property ratio; changing speed alone does not change it or determine its value from Re.

The plate derivation uses $\mathrm{Re}_x = u_\infty x/\nu$, $\mathrm{Pr} = \nu/\alpha$ and $\mathrm{Nu}_x = h_x x/k$, with $\alpha = k/(\rho C_p)$ and $\nu = \mu/\rho$.

QUIZ
1. Heat crossing the wall-adjacent fluid
Air flows over a stationary, impermeable hot wall with no slip. How does heat cross the fluid immediately next to the wall?

1. Only by radiation.
2. By fluid flowing through the wall.
3. By conduction through the fluid next to the wall.
4. It cannot cross because the wall velocity is zero.

See solution for Quiz 1

QUIZ

2. The conductivity in Nusselt number

Which conductivity belongs in $\mathrm{Nu}_L = hL/k$ for air flowing over a metal plate?

1. The air conductivity.
2. The plate conductivity.
3. The sum of air and plate conductivities.
4. The larger of the two conductivities.

See solution for Quiz 2

4.1 Boundary layer approach: flow over a flat plate

DEFINITION

Velocity boundary layer

The region next to a surface where viscous effects make the fluid velocity differ appreciably from the outer flow. Its conventional edge is not a sharp physical boundary.

DEFINITION

Thermal boundary layer

The region next to a surface where the fluid temperature differs appreciably from the outer fluid temperature. Its thickness depends on heat diffusion and fluid motion.

Consider a smooth plate aligned with a uniform free stream. Let x run from the leading edge and y point normally into the fluid. Write u_x and u_y for the velocity components. No slip gives $u_x = u_y = 0$ at the wall. The conventional velocity thickness δ is where u_x reaches 99 percent of u_∞. For uniform surface temperature the thermal thickness δ_t is where

$$\frac{T - T_s}{T_\infty - T_s} = 0.99.$$

Neither layer has a sharp physical edge. Both grow downstream. The temperature drop concentrated near the wall explains why its layer provides much of the thermal resistance. The estimate $h \sim k/\delta_t$ says that a thinner thermal layer gives larger h.

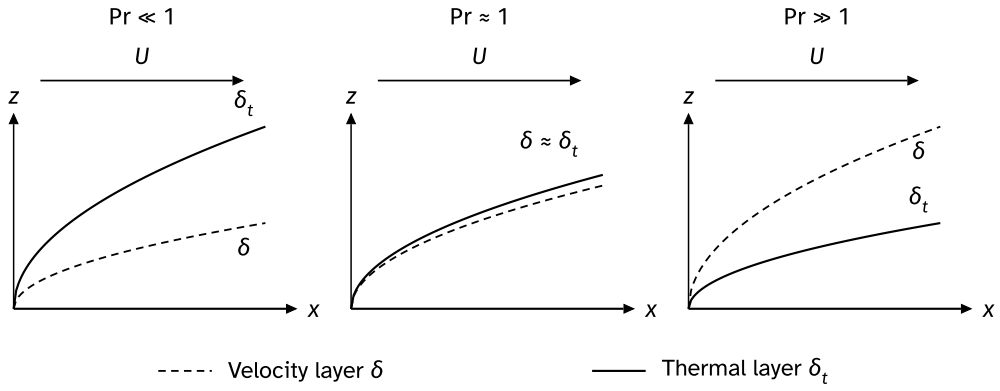


Figure 1: Three schematic plate boundary layers: heat spreads farther than momentum at small Prandtl number, equally near one, and less far at large Prandtl number.

Relative boundary-layer thicknesses for small, near-unit and large Prandtl numbers. The curves are schematic.

Use $\mathrm{Re}_c = 5 \times 10^5$ as the nominal transition estimate.

Assume steady, incompressible, two-dimensional laminar flow, constant properties, negligible viscous heating and no heat generation. The plate is isothermal, and its outer velocity is constant, so the pressure gradient is zero. In a thin layer, normal gradients dominate streamwise diffusion. The continuity, momentum and thermal energy equations reduce to

$$\begin{aligned} \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} &= 0 \\ u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} &= \nu \frac{\partial^2 u_x}{\partial y^2} \end{aligned} \tag{2}$$

$$u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

At the wall $T = T_s$; far outside the layer $u_x = u_\infty$ and $T = T_\infty$. For Pr equal to one the diffusivities coincide. The normalised fields

$$\frac{u_x}{u_\infty} \qquad \text{and} \qquad \frac{T - T_s}{T_\infty - T_s}$$

then obey the same equation and boundary values. This is an exact analogy within the reduced boundary-layer model, not an exact solution of the full Navier-Stokes system.

Blasius uses

$$\eta = y \sqrt{\frac{u_\infty}{\nu x}}$$

and a streamfunction

$$\psi = \sqrt{\nu u_\infty x}\, f(\eta),$$

so $u_x = u_\infty f'$. The momentum equation becomes

$$f''' + \frac{1}{2} f f'' = 0,$$

with boundary conditions

$$f(0) = f'(0) = 0, \qquad f'(\infty) = 1.$$

Its numerical solution gives $f''(0) = 0.33206$. Thus for Pr equal to one,

$$\begin{aligned} \left[\frac{\partial u_x}{\partial y}\right]_{y=0} &= 0.332 \frac{u_\infty}{x} \mathrm{Re}_x^{1/2} \\ \left[\frac{\partial T}{\partial y}\right]_{y=0} &= 0.332 \frac{T_\infty - T_s}{x} \mathrm{Re}_x^{1/2} \end{aligned} \tag{4}$$

Equating $-k\partial T/\partial y$ at the wall to Eq. 1 cancels the temperature difference. Pohlhausen's thermal treatment supplies the approximately $\mathrm{Pr}^{1/3}$ dependence for ordinary fluids:

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332 \mathrm{Re}_x^{1/2} \mathrm{Pr}^{1/3} \tag{5}$$

The approximate ratio of boundary-layer thicknesses is

$$\frac{\delta}{\delta_t} \simeq \mathrm{Pr}^{1/3}$$

The correlation applies to an isothermal, zero-pressure-gradient laminar plate, usually with $\mathrm{Pr} \gtrsim 0.6$. The thickness relation is an approximation, not an identity valid for all Pr. Liquid metals need a separate low-Pr treatment. The gradient is normal to the wall, in y, even where the source prints x. For Pr equal to one Eq. 5 reduces to the preceding gradient result. A uniform heat-flux plate has a different coefficient and cannot use this isothermal correlation unchanged.

QUIZ

3. Boundary-layer thickness

For the stated laminar flat-plate approximation with $\mathrm{Pr} > 1$, how do the thermal and velocity boundary-layer thicknesses compare?

1. The thermal layer is thicker because heat diffuses faster than momentum.
2. Both layers must have identical thickness.
3. Prandtl number determines only the transition position.
4. The thermal layer is thinner because heat diffuses more slowly than momentum.

See solution for Quiz 3

4.2 Local and mean coefficients

A local coefficient predicts flux at one position. Total heat transfer needs an integral. For width w and uniform surface temperature,

$$\dot{Q} = w(T_s - T_\infty)\int_0^L h_x\,dx$$

$$\bar{h} = \frac{1}{L}\int_0^L h_x\,dx \tag{6}$$

In Eq. 5,

$$h_x = 0.332k\mathrm{Pr}^{1/3}\sqrt{\frac{u_\infty}{\nu}}\,x^{-1/2}.$$

Since the integral of $x^{-1/2}$ from zero to L is $2\sqrt{L}$,

$$\begin{aligned}\overline{\mathrm{Nu}}_L &= \frac{\bar{h}L}{k} = 0.664\mathrm{Re}_L^{1/2}\mathrm{Pr}^{1/3}\\ \bar{h} &= 2[h_x]_{x=L}\end{aligned} \tag{7}$$

The formal local singularity at $x = 0$ is integrable. The thin-layer approximation breaks down sufficiently near the leading edge, so it does not predict a physically infinite coefficient there. Evaluate plate properties at the film temperature

$$T_f = \frac{T_s + T_\infty}{2} \tag{8}$$

If T_s is unknown, guess it, evaluate properties, solve the heat balance, and repeat until the assumed and calculated temperatures agree.

QUIZ

4. Local versus average transfer

An isothermal plate stays laminar over its whole length. If the trailing-edge coefficient is $12\ \mathrm{W\,m^{-2}\,K^{-1}}$, what is its length-average coefficient under the stated correlation?

1. $6\ \mathrm{W\,m^{-2}\,K^{-1}}$.
2. $24\ \mathrm{W\,m^{-2}\,K^{-1}}$.
3. $12\ \mathrm{W\,m^{-2}\,K^{-1}}$.
4. $48\ \mathrm{W\,m^{-2}\,K^{-1}}$.

See solution for Quiz 4

4.3 Turbulent flow and transition

Write instantaneous temperature as $T = \overline{T} + T'$ with $\overline{T'} = 0$. Averaging the energy equation introduces the turbulent heat flux $\rho C_p\overline{u_y'T'}$. An eddy diffusivity α_t models it, giving normal total heat flux

$$-\rho C_p(\alpha + \alpha_t)\frac{\partial \overline{T}}{\partial y}.$$

Unlike α, α_t belongs to the flow and changes with position. Eddies transport heat across the layer, but vanish at the wall. Molecular transport still limits the wall flux. A closure model or measurements are needed; Blasius does not solve turbulence.

The source's turbulent local form and its integrated average are

$$\begin{aligned} \mathrm{Nu}_x &= 0.0296\mathrm{Re}_x^{0.8}\mathrm{Pr}^{1/3} \\ \overline{\mathrm{Nu}}_L &= 0.037\mathrm{Re}_L^{0.8}\mathrm{Pr}^{1/3} \end{aligned} \tag{9}$$

The average assumes turbulence from the leading edge. The factor 0.037 is $0.0296/0.8$, from integrating $x^{-0.2}$. The source uses the close empirical variant 0.0366; the notes use the internally consistent integral. Typical conditions are $5 \times 10^5 < \mathrm{Re} < 10^7$ and ordinary-fluid Pr, roughly 0.6 to 60. For a plate with a laminar starting length, integrate Eq. 6 in two pieces. Define

$$B = 0.664\mathrm{Re}_c^{1/2} - 0.037\mathrm{Re}_c^{0.8}$$

to obtain

$$\overline{\mathrm{Nu}}_L = [0.037\mathrm{Re}_L^{0.8} + B]\mathrm{Pr}^{1/3} \tag{10}$$

Here $\mathrm{Re}_L > \mathrm{Re}_c$. At $\mathrm{Re}_L = 10^6$, ignoring the laminar run gives a substantially larger answer. The subtraction becomes a small correction at large Re. The source describes the difference as 37 percent: that is the reduction relative to the all-turbulent estimate, not the overprediction percentage relative to the combined result.

4.4 Reynolds and Colburn analogies

DEFINITION

Skin friction coefficient

The dimensionless wall shear

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2},$$

using the specified reference speed U. A local coefficient uses local wall shear; an average coefficient uses its surface average. It excludes pressure drag.

The wall shear stress τ_w in Pa and local skin friction coefficient $C_{f,x}$ are defined by

$$\tau_w = \mu\left[\frac{\partial u_x}{\partial y}\right]_{y=0} = \frac{1}{2}C_{f,x}\rho u_\infty^2 \tag{11}$$

DEFINITION
Stanton number

The thermal Stanton number is

$$\mathrm{St} = \frac{h}{\rho U C_p} = \frac{\mathrm{Nu}}{\mathrm{Re}\,\mathrm{Pr}},$$

using the same speed and length in all groups. It compares the surface heat-transfer coefficient with the fluid's heat-capacity flux per unit area. Use consistent local or average quantities.

Small St means a small heat flux relative to $\rho U C_p \Delta T$; larger St means stronger surface exchange relative to that flowing capacity. For $h = 20$, $\rho = 1$, $U = 10$ and $C_p = 1000$ in SI units, $\mathrm{St} = 0.002$. This is not the fraction of a whole stream's temperature change, which also depends on area and flow rate through the internal-flow balance. The definition does not require the Reynolds analogy to hold.

At Pr equal to one, the equal normalised gradients in Eq. 4 and $\mu C_p = k$ give $h_x = C_p \tau_w / u_\infty$. Division by the stream heat-capacity flux gives the Reynolds analogy:

$$\mathrm{St}_x = \frac{h_x}{\rho u_\infty C_p} = \frac{\mathrm{Nu}_x}{\mathrm{Re}_x \mathrm{Pr}} = \frac{C_{f,x}}{2} \tag{12}$$

DEFINITION
Drag coefficient

The dimensionless total drag

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A_{\mathrm{ref}}},$$

using the specified reference speed and area. For a bluff body the reference area is commonly its projected area normal to the flow. Total drag includes both wall shear and pressure forces; it is not a skin friction coefficient.

St is the dimensionless Stanton number. This relation requires Pr equal to one and negligible pressure-gradient effects. Use skin friction, not total drag on a bluff body: pressure drag has no corresponding wall heat flux.

DEFINITION
Colburn heat factor

The dimensionless factor $j_H = \mathrm{St}\,\mathrm{Pr}^{2/3}$. It combines heat transfer with a property-ratio correction. For suitable flows it is approximately $C_f/2$; that analogy is an additional model, not its definition.

For ordinary Pr the Colburn form is

$$
\begin{aligned}
j_H &= \mathrm{St}_x \mathrm{Pr}^{2/3} = \frac{C_{f,x}}{2} \\
\mathrm{Nu}_x &= \frac{C_{f,x}}{2} \mathrm{Re}_x \mathrm{Pr}^{1/3}
\end{aligned}
\tag{13}
$$

The Pr exponent follows algebraically from St's definition and the $\mathrm{Pr}^{1/3}$ thermal result. With

$$C_{f,x} = 0.664 \mathrm{Re}_x^{-1/2}$$

it reproduces Eq. 5; with

$$C_{f,x} = 0.0592 \mathrm{Re}_x^{-1/5}$$

it gives Eq. 9. Use matching local coefficients, or consistently averaged ones. The usual range is approximately $0.6 < \mathrm{Pr} < 60$ for smooth plates without form drag. Raising heat transfer often increases friction and pumping demand, but the analogy alone does not prove a universal economic rule for equipment.

EXERCISE

Exercise 1. Low-pressure plate

Air at a pressure of $6\ \mathrm{kN/m^2}$ and temperature of $300\ ^\circ\mathrm{C}$ flows with a velocity of $10\ \mathrm{m/s}$ over a flat plate 0.5 m long. Estimate the cooling rate per unit width of the plate needed to maintain the surface at a temperature of $27\ ^\circ\mathrm{C}$.

- $\nu_{@1\ \mathrm{atm}} = 30.84 \times 10^{-6}\ \mathrm{m^2/s}$
- $k_{\text{air at } 165\,^\circ\mathrm{C}} = 3.6427 \times 10^{-2}\ \mathrm{W/(m \cdot K)}$
- $\mathrm{Pr}_{\text{air at } 165\,^\circ\mathrm{C}} \simeq 0.684$

See solution for Exercise 1

4.5 Boundary layer approach: flow over a sphere

First consider the zero-flow limit. A sphere of radius $R = D/2$ has uniform surface temperature T_R. An infinite stagnant fluid has far-field temperature T_∞ and constant k. Assume steady spherical symmetry, no generation and no buoyancy-induced flow. Starting from

$$\rho C_p \frac{DT}{Dt} = \nabla \cdot (k \nabla T) + S,$$

stationarity removes time dependence, stagnation removes advection, symmetry removes angular gradients and $S = 0$ removes generation. Only the radial term remains:

$$
\begin{aligned}
\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) &= 0 \\
T(R) = T_R, \quad T(\infty) &= T_\infty
\end{aligned}
\tag{16}
$$

The first integration gives

$$r^2 \frac{dT}{dr} = C_1,$$

not zero as printed on the source slide. The second gives

$$T = -\frac{C_1}{r} + C_2.$$

The far-field condition gives $C_2 = T_\infty$ and the surface gives $C_1 = R(T_\infty - T_R)$. Thus

$$T = T_\infty + \frac{R}{r}(T_R - T_\infty) \tag{17}$$

The full spherical energy equation's polar diffusion term is

$$(r^2 \sin\vartheta)^{-1} \partial_\vartheta (k \sin\vartheta \, \partial_\vartheta T).$$

The source omits its inner sine; symmetry makes that term zero here. ϑ is a polar angle, distinct from the dimensional excess temperature θ used in transient conduction.

Differentiating Eq. 17 and equating the outward flux to Newton's definition yields

$$q_R = \frac{k(T_R - T_\infty)}{R}$$

$$h = \frac{k}{R} = \frac{2k}{D}$$

$$\mathrm{Nu}_D = 2 \tag{18}$$

The exact conduction resistance to infinity is $1/(4\pi k R)$. The analogous integrals for an infinite cylinder and plane diverge. Hence their infinite-medium steady problems do not share this finite conduction limit. For a 1 mm droplet in air with $k = 0.026$, the stagnant model gives $h = 52$ in SI units; for 10 micrometres it gives 5200. For equal-density spherical particles the area per mass is $6/(\rho_p D)$, so halving diameter doubles both h and area per mass, quadrupling their product.

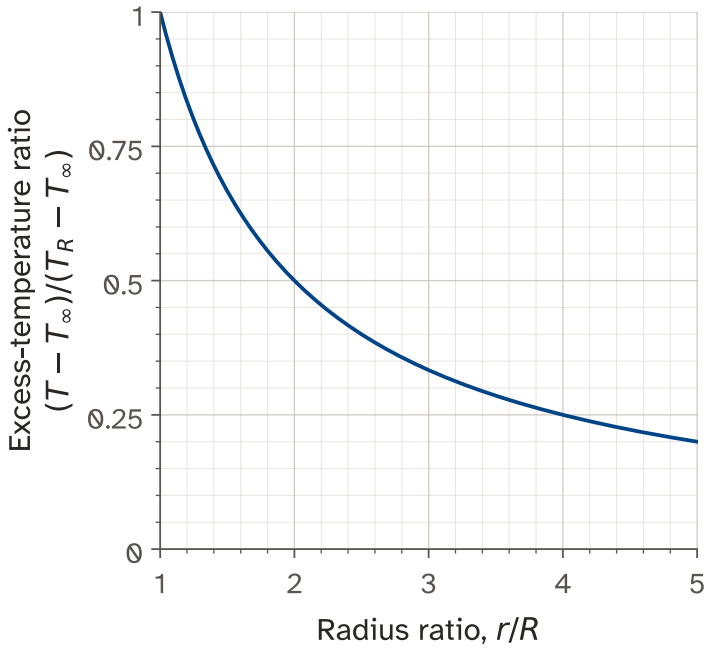


Figure 3: Calculated stagnant-sphere excess temperature drops from one at radius R to one half at 2R and one quarter at 4R.

Original calculated profile R/r from Eq. 17, following Bird et al. (2006) and Çengel and Ghajar (2015). The curve approaches the far field asymptotically; there is no finite outer edge.

QUIZ

5. The stagnant-sphere limit

A sphere transfers heat by steady conduction into an infinite stagnant fluid, with buoyancy excluded. What is its Nusselt number based on diameter?

1. 0.
2. 1.
3. 2.
4. It must be infinite.

See solution for Quiz 5

4.6 Forced flow around a sphere

A moving fluid adds convection to the stagnant conduction limit. Use the relative speed between the sphere and the undisturbed fluid. Both correlations below give a surface-average coefficient for an isolated sphere, based on its diameter D and area πD^2. They do not describe a packed bed.

For a small sphere with modest property variation, the Ranz-Marshall estimate is

$$\overline{\mathrm{Nu}}_D = 2 + 0.6\mathrm{Re}_D^{1/2}\mathrm{Pr}^{1/3} \tag{19}$$

Use the low-Re range $0 < \mathrm{Re}_D \lesssim 200$. For the gas examples here, restrict use to $0.6 \leq \mathrm{Pr} \leq 1$, near air's value of 0.7. Check a fluid-specific range before applying it to other fluids. Eval-

uate the supplied properties at the film temperature $T_f = (T_s + T_\infty)/2$. Strong evaporation, deformation or large property changes need a more suitable model. The 2 recovers Eq. 18 as relative speed tends to zero, with buoyancy excluded.

For a wider range, the Whitaker correlation is

$$\begin{gathered}\overline{\mathrm{Nu}}_D = 2 + \left(0.4\mathrm{Re}_D^{1/2} + 0.06\mathrm{Re}_D^{2/3}\right)\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \\ 3.5 \le \mathrm{Re}_D \le 7.6 \times 10^4, \qquad 0.71 \le \mathrm{Pr} \le 380 \\ 1 \le \frac{\mu_\infty}{\mu_s} \le 3.2\end{gathered} \tag{20}$$

Evaluate ρ, μ_∞, k, C_p, Reynolds number and Prandtl number at the approaching-stream temperature T_∞. Evaluate only μ_s at T_s. This differs from the film-temperature rule above. Recover $\bar{h} = \overline{\mathrm{Nu}}_D k_\infty / D$ with the same conductivity basis. These are the conditions of the Whitaker correlation, also documented in the Ansys particle heat-transfer manual.

4.6.1 Example: selecting a sphere correlation

An isolated 10 mm sphere sees a relative air speed of 0.10 m s^{-1}. For a modest surface-to-air temperature difference, take film properties $\rho = 1.18\,\mathrm{kg\,m^{-3}}$, $\mu = 1.85 \times 10^{-5}\,\mathrm{Pa\,s}$, $k = 0.0263\,\mathrm{W\,m^{-1}\,K^{-1}}$ and $\mathrm{Pr} = 0.71$. Then $\mathrm{Re}_D = 63.8$, so Ranz-Marshall is within its low-Re range:

$$\begin{aligned}\overline{\mathrm{Nu}}_D &= 2 + 0.6(63.8)^{1/2}(0.71)^{1/3} = 6.27 \\ \bar{h} &= \frac{6.27(0.0263)}{0.010} \simeq 16.5\,\mathrm{W\,m^{-2}\,K^{-1}}.\end{aligned}$$

At $\mathrm{Re}_D = 1000$, select Whitaker if its other conditions are met; Ranz-Marshall is outside the range used here. Recalculate properties using Whitaker's approaching-stream rule before evaluating it.

4.7 Cylinder cross-flow correlations

For the cylinder calculation below, use the slide's average correlation

$$\overline{\mathrm{Nu}}_D = C\mathrm{Re}_D^m \mathrm{Pr}^{1/3} \tag{21}$$

For a circular cylinder at $4000 < \mathrm{Re}_D < 40000$, $C = 0.193$ and $m = 0.618$. Evaluate properties at film temperature and use D in both Re and Nu. For other geometries or ranges, use the matching slide correlation and its stated limits.

Circular-cylinder Re interval	C	m
4000 to 40000	0.193	0.618

Table 1: Coefficients used in classroom example 2, following the source deck and Çengel and Ghajar (2015), Table 7-1. For the comparison in the solution, Churchill and Bernstein give

$$\overline{\mathrm{Nu}}_D = 0.3 + \frac{0.62 \mathrm{Re}_D^{1/2} \mathrm{Pr}^{1/3}}{G} H$$
$$G = \left[1 + \left(\frac{0.4}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4} \tag{22}$$
$$H = \left[1 + \left(\frac{\mathrm{Re}_D}{282000}\right)^{5/8}\right]^{4/5}$$

Use film-temperature properties and $\mathrm{Re}_D \mathrm{Pr} > 0.2$ for an isolated long circular cylinder in cross flow.

EXERCISE

Exercise 2. Heated cylinder measurement

A metallic cylinder (12.7 mm in diameter and 94 mm long) is heated internally by an electrical heater and is subjected to a cross flow of air. The air velocity and temperature are maintained at $10\ \mathrm{m/s}$ and $26.2\ ^\circ\mathrm{C}$. The heater power dissipation is measured to be 39.1 W, while the average cylinder surface temperature is $128.4\ ^\circ\mathrm{C}$. Compare the heat transfer coefficient from experimental observations and correlation predictions.

See solution for Exercise 2

QUIZ

6. Selecting a correlation

A smooth plate has a laminar leading region followed by a turbulent region. You need its total heat rate. Which treatment matches that flow?

1. Use an average correlation that includes both the laminar and turbulent regions.
2. Use the turbulent local coefficient at the trailing edge over the whole plate.
3. Use the sphere correlation because the fluid is the same.
4. Choose the correlation with the largest Nusselt number.

See solution for Quiz 6

5 Additional examples

EXERCISE

Exercise 3. Heat transfer from a skin-friction measurement

Air flows along a smooth flat plate at 15 m s^{-1}. The measured average skin-friction coefficient is $\bar{C}_f = 0.0040$. At the film temperature, use $\rho = 1.18$ kg m^{-3}, $C_p = 1005$ J kg^{-1} K^{-1} and $\mathrm{Pr} = 0.72$. Estimate the average heat transfer coefficient with the Colburn analogy. Explain why the uncorrected Reynolds analogy gives a different answer. Assume matching flow and thermal conditions for the analogy and negligible pressure drag.

See solution for Exercise 3

EXERCISE

Exercise 4. Where does a local plate coefficient fall to half its value?

Air flows at 0.80 m s^{-1} over an isothermal flat plate. In the laminar region, the local coefficient at $x_1 = 0.25$ m is $h_{x_1} = 12.0$ W m^{-2} K^{-1}. Use $\nu = 1.60 \times 10^{-5}$ m^2 s^{-1} and constant properties. At what distance will the local coefficient be 6.0 W m^{-2} K^{-1}? Find the average coefficient between the leading edge and that distance, and check the laminar assumption.

See solution for Exercise 4

6 What you should now be able to calculate

Select a local or average coefficient for a plate, sphere or cylinder, check the correlation limits, and calculate a heat rate with the matching area.

Practice: Workshop 7 for forced flow over a pipe; Workshop 11 for analogous convective mass transfer.

7 Report an error

Email Ranjeet Utikar with the title Forced convection in external flow, the URL `/heat-transport/external-forced-convection/`, and the equation or passage to check.

References

Bird, R. Byron, Warren E. Stewart, and Edwin N. Lightfoot. 2006. *Transport Phenomena.* Revised 2nd. John Wiley & Sons. https://www.wiley-vch.de/en/areas-interest/engineering/transport-phenomena-978-0-470-11539-8.

Çengel, Yunus A., and Afshin J. Ghajar. 2015. *Heat and Mass Transfer: Fundamentals and Applications.* 5th ed. McGraw-Hill Education. https://glencoe.mheducation.com/sites/0073398187/information_center_view0/application_chapters.html.

Shah, Milinkumar. n.d. “CHEN3001-TP: Heat Transport 3.” Curtin University. Unpublished.

Solutions

Quiz solutions

Quiz 1. Heat crossing the wall-adjacent fluid

Correct choice: 3.

The fluid velocity is zero at a stationary no-slip wall, but its temperature gradient is not. Conduction carries heat across the adjacent fluid; motion farther away changes the temperature field and maintains the gradient.

Back to Quiz 1

Quiz 2. The conductivity in Nusselt number

Correct choice: 1.

Nusselt number describes transfer through the fluid boundary layer and uses fluid conductivity. The solid conductivity belongs in a Biot number when assessing temperature gradients inside the plate.

Back to Quiz 2

Quiz 3. Boundary-layer thickness

Correct choice: 4.

The Prandtl number compares momentum and heat diffusion:

$$\mathrm{Pr} = \frac{\nu}{\alpha} > 1$$

Momentum spreads farther than heat. Within the stated flat-plate approximation,

$$\frac{\delta}{\delta_t} \simeq \mathrm{Pr}^{1/3} > 1$$

The thermal boundary layer is therefore thinner than the velocity boundary layer.

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Quiz 4. Local versus average transfer

Correct choice: 2.

For this laminar, constant-temperature plate,

$$h_x \propto x^{-1/2}$$

The coefficient is larger near the leading edge. Averaging along the plate gives

$$\bar{h} = 2h_L = 2(12) = 24\ \mathrm{W\,m^{-2}\,K^{-1}}$$

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Quiz 5. The stagnant-sphere limit

Correct choice: 3.

The spherical conduction solution gives

$$h = \frac{k}{R} = \frac{2k}{D}$$

Using the diameter in Nusselt number,

$$\mathrm{Nu}_D = \frac{hD}{k} = 2$$

Heat can leave the sphere by conduction even when the fluid does not move.

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Quiz 6. Selecting a correlation

Correct choice: 1.

Total heat rate needs an average over the actual plate. A combined relation integrates the laminar and turbulent portions. Geometry, wall condition, property rule and validity range must also match the chosen correlation.

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Exercise solutions

Exercise 1. Low-pressure plate

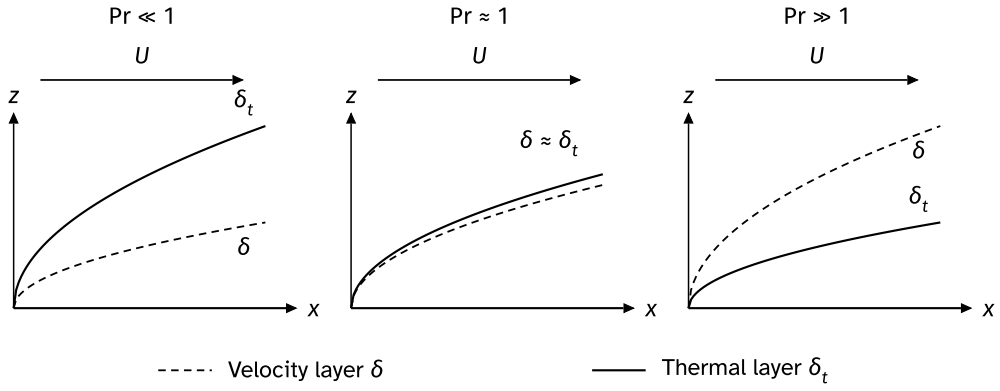


Figure 2: Air flows along the plate, and the velocity and thermal boundary layers grow from its leading edge. Use the full plate length for the average coefficient.

The plate geometry is the boundary-layer geometry in Figure 1. The stream runs along the 0.50 m length, and the cooling rate is requested per metre of width.

Data

- Air temperature $T_\infty = 300\,^\circ\mathrm{C}$
- Plate temperature $T_s = 27\,^\circ\mathrm{C}$
- Plate length $L = 0.50\,\mathrm{m}$
- Air speed $u_\infty = 10\,\mathrm{m\,s^{-1}}$
- Absolute pressure $p = 6\,\mathrm{kPa}$

The supplied conductivity, Prandtl number and one-atmosphere viscosity are evaluated near the film temperature.

Assumptions

- Steady state.
- Smooth plate aligned with the free stream.
- Constant fluid properties at the film temperature.
- Ideal-gas density and pressure-independent dynamic viscosity.
- Radiation and buoyancy effects are neglected.

Calculations

The hot air heats the colder plate. The required cooling rate is the heat received by one exposed plate face. First evaluate the film temperature:

$$T_f = \frac{T_s + T_\infty}{2} = \frac{27 + 300}{2} = 163.5\,^\circ\mathrm{C}$$

The supplied properties at $165\,^\circ\mathrm{C}$ approximate this temperature. At fixed temperature, ideal-gas density is proportional to absolute pressure. Since $\nu = \mu/\rho$ and the dynamic viscosity is nearly pressure independent, the kinematic viscosity increases as pressure falls:

$$\nu = \nu_{1\,\mathrm{atm}} \frac{p_{1\,\mathrm{atm}}}{p} = 30.84 \times 10^{-6} \frac{101.325}{6} = 5.2081 \times 10^{-4}\ \mathrm{m^2\,s^{-1}} \tag{14}$$

The Reynolds number based on the full plate length determines the flow regime:

$$\mathrm{Re}_L = \frac{u_\infty L}{\nu} = \frac{10(0.50)}{5.2081 \times 10^{-4}} = 9600.4$$

This is well below the nominal transition value 5×10^5. Use the laminar, isothermal-plate average in Eq. 7:

$$\overline{\mathrm{Nu}}_L = 0.664 \mathrm{Re}_L^{1/2} \mathrm{Pr}^{1/3} = 0.664(9600.4)^{1/2}(0.684)^{1/3} = 57.323$$

Recover the dimensional coefficient using the same length:

$$\bar{h} = \frac{k \overline{\mathrm{Nu}}_L}{L} = \frac{(3.6427 \times 10^{-2})(57.323)}{0.50} = 4.1762\ \mathrm{W\,m^{-2}\,K^{-1}}$$

For plate width w, the exposed area is Lw. Newton's relation therefore gives

$$\frac{\dot{Q}_{\mathrm{removed}}}{w} = \bar{h} L (T_\infty - T_s) = 4.1762(0.50)(300 - 27) = 570.1\ \mathrm{W\,m^{-1}} \tag{15}$$

As a geometry check, the laminar thickness estimate gives

$$\delta(L) \simeq \frac{5L}{\sqrt{\mathrm{Re}_L}} = 0.0255\ \mathrm{m},$$

about five percent of the plate length. Ignoring the low-pressure viscosity correction would overestimate h by a factor of about 4.11. The estimated molecular mean free path is much smaller than this boundary-layer thickness, so the low pressure does not by itself invalidate the continuum model.

Answer

The cooling system must remove approximately 570 W per metre of plate width from the exposed face.

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Exercise 2. Heated cylinder measurement

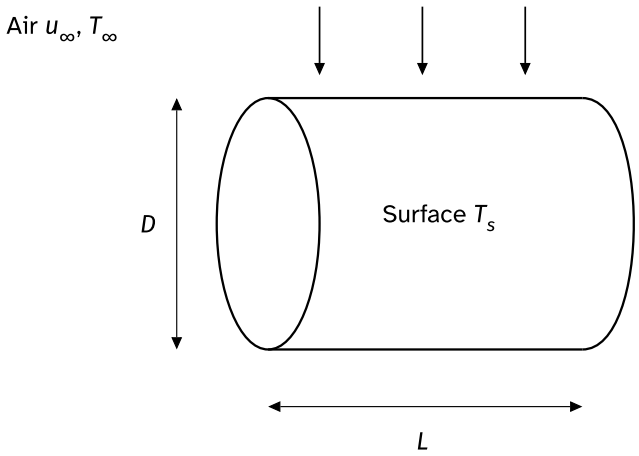


Figure 4: Air flows across the cylinder, perpendicular to its axis. The convecting lateral area is pi D L when the end faces are neglected.

Data

Quantity	Value
Cylinder diameter	$D = 0.0127\ \mathrm{m}$
Cylinder length	$L = 0.094\ \mathrm{m}$
Air speed	$u_\infty = 10\ \mathrm{m\,s^{-1}}$
Air temperature	$T_\infty = 26.2\ ^\circ\mathrm{C}$
Surface temperature	$T_s = 128.4\ ^\circ\mathrm{C}$
Measured heat dissipation	$\dot{Q} = 39.1\ \mathrm{W}$
Air density	$\rho = 0.995\ \mathrm{kg\,m^{-3}}$
Dynamic viscosity	$\mu = 2.082 \times 10^{-5}\ \mathrm{Pa\,s}$
Air conductivity	$k = 0.030\ \mathrm{W\,m^{-1}\,K^{-1}}$
Prandtl number	$\mathrm{Pr} = 0.700$

The film temperature is the mean of the surface and air temperatures, 77.3 degrees Celsius or 350.45 K. Use the lecturer’s property set near 350 K.

Assumptions

- Steady state.
- Uniform mean surface temperature.
- Constant fluid properties at the film temperature.
- The stated 39.1 W leaves through the curved cylinder surface.
- Radiation and heat transfer through the ends are neglected in the comparison.

- The long-cylinder cross-flow correlations approximate the finite cylinder.

Calculations

The measured coefficient follows from Newton's relation. The curved area is

$$A = \pi DL = \pi(0.0127)(0.094) = 0.00375043\ \mathrm{m}^2,$$

giving

$$h_{\mathrm{exp}} = \frac{\dot{Q}}{A(T_s - T_\infty)} = \frac{39.1}{0.00375043(128.4 - 26.2)} = 102.01\ \mathrm{W\,m^{-2}\,K^{-1}} \tag{23}$$

The flow correlation uses diameter, rather than cylinder length:

$$\mathrm{Re}_D = \frac{\rho u_\infty D}{\mu} = \frac{0.995(10)(0.0127)}{2.082 \times 10^{-5}} = 6069.4$$

This lies in the interval $4000 < \mathrm{Re}_D < 40000$, so Table 1 specifies $C = 0.193$ and $m = 0.618$. The power law in Eq. 21 gives

$$\overline{\mathrm{Nu}}_D = C\mathrm{Re}_D^m \mathrm{Pr}^{1/3} = 0.193(6069.4)^{0.618}(0.700)^{1/3} = 37.317$$

The corresponding coefficient is

$$\bar{h} = \frac{k\overline{\mathrm{Nu}}_D}{D} = \frac{0.030(37.317)}{0.0127} = 88.15\ \mathrm{W\,m^{-2}\,K^{-1}}$$

For an independent correlation comparison, substitute the same Re and Pr into the Churchill-Bernstein relation in Eq. 22:

$$\overline{\mathrm{Nu}}_D = 0.3 + \frac{0.62(6069.4)^{1/2}(0.700)^{1/3}}{\left[1 + \left(\frac{0.4}{0.700}\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{6069.4}{282000}\right)^{5/8}\right]^{4/5} = 40.631$$

Converting this prediction to a coefficient gives

$$\bar{h}_{\mathrm{CB}} = \frac{0.030(40.631)}{0.0127} = 95.98\ \mathrm{W\,m^{-2}\,K^{-1}}$$

The power law is 13.6 percent below the measured coefficient; Churchill and Bernstein is 5.9 percent below it. The experimental value is an effective coefficient under the assumed heat balance. The original slide diagram also shows 46 W heater input and 6.9 W losses, whose difference is 39.1 W. Using the full 46 W with the curved area would include heat leaving by another route.

Radiation would reduce the heat assigned to convection. For illustration, $\epsilon = 0.5$ and surroundings at the air temperature give

$$\begin{aligned}\dot{Q}_{\mathrm{rad}} &= \epsilon\sigma A(T_s^4 - T_{\mathrm{surr}}^4) \\ &= 0.5(5.6704 \times 10^{-8})(0.00375043)[(401.55)^4 - (299.35)^4] = 1.91\ \mathrm{W}\end{aligned}$$

Subtracting this illustrative loss gives a convection-only measured coefficient of about $97.0\ \mathrm{W\,m^{-2}\,K^{-1}}$. The emissivity is not supplied, so this is a sensitivity check. The finite aspect ratio $L/D = 7.40$ and uncertain support losses also limit the comparison.

Answer

The measured effective coefficient is approximately $102\ \mathrm{W\,m^{-2}\,K^{-1}}$. The cylinder power law predicts $88.2\ \mathrm{W\,m^{-2}\,K^{-1}}$ and Churchill-Bernstein predicts $96.0\ \mathrm{W\,m^{-2}\,K^{-1}}$ under the stated assumptions.

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Exercise 3. Heat transfer from a skin-friction measurement

Data

- Air speed $u_\infty = 15\ \mathrm{m\,s^{-1}}$
- Average skin-friction coefficient $\bar{C}_f = 0.0040$
- Film density $\rho = 1.18\ \mathrm{kg\,m^{-3}}$
- Specific heat capacity $C_p = 1005\ \mathrm{J\,kg^{-1}\,K^{-1}}$
- Prandtl number $\mathrm{Pr} = 0.72$

Assumptions

- Matching flow and thermal conditions for the Colburn analogy.
- Negligible pressure drag.
- Constant fluid properties.

Calculations

Use average quantities throughout. Equation Eq. 13 gives

$$\overline{\mathrm{St}} = \frac{\bar{C}_f}{2}\mathrm{Pr}^{-2/3} = 0.0020(0.72)^{-2/3} = 0.002490$$

Recover the dimensional coefficient from Stanton number:

$$\bar{h} = \overline{\mathrm{St}}\rho u_\infty C_p = 0.002490(1.18)(15)(1005) = 44.3\ \mathrm{W\,m^{-2}\,K^{-1}}$$

The plain Reynolds analogy would set $\overline{\mathrm{St}} = 0.0020$ and give 35.6 W m^{-2} K^{-1}. It assumes Pr equal to one. At the supplied Pr, the Colburn correction increases the estimate by a factor of 1.245.

No viscosity or plate length is required because skin friction is measured. Using a local friction coefficient with an average heat transfer coefficient would mix two different quantities. Total drag on a sphere or cylinder also cannot replace the plate skin-friction coefficient in this calculation.

Answer

The Colburn estimate is $\bar{h} = 44.3\ \mathrm{W\,m^{-2}\,K^{-1}}$. The uncorrected Reynolds analogy gives $35.6\ \mathrm{W\,m^{-2}\,K^{-1}}$, because it omits the Prandtl-number correction.

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Exercise 4. Where does a local plate coefficient fall to half its value?

Data

- Air speed $u_\infty = 0.80\ \mathrm{m\,s^{-1}}$
- First position $x_1 = 0.25\ \mathrm{m}$
- Coefficient at the first position $h_{x_1} = 12.0\ \mathrm{W\,m^{-2}\,K^{-1}}$
- Target coefficient $h_{x_2} = 6.0\ \mathrm{W\,m^{-2}\,K^{-1}}$
- Kinematic viscosity $\nu = 1.60 \times 10^{-5}\ \mathrm{m^2\,s^{-1}}$

Assumptions

- Steady laminar flow over an isothermal plate.
- Constant fluid properties.

Calculations

For fixed speed and properties, the local relation has $\mathrm{Nu}_x \propto \mathrm{Re}_x^{1/2} \propto x^{1/2}$. Because $h_x = \mathrm{Nu}_x k/x$, it follows that $h_x \propto x^{-1/2}$:

$$\frac{h_{x_2}}{h_{x_1}} = \left(\frac{x_1}{x_2}\right)^{1/2}$$

$$x_2 = x_1\left(\frac{12.0}{6.0}\right)^2 = 1.00\ \mathrm{m}$$

Halving a local coefficient requires four times the distance, not twice. The layer thickens downstream and the wall temperature gradient decreases.

The length average is a different quantity. Integrating the local law gives

$$\bar{h}_L = \frac{1}{L}\int_0^L Cx^{-1/2}\,dx = 2CL^{-1/2} = 2h_L$$

Thus the average over the first 1.00 m is 12.0 W m⁻² K⁻¹ even though the local coefficient at its trailing edge is 6.0 W m⁻² K⁻¹. The ideal leading-edge singularity is integrable.

At x_2,

$$\mathrm{Re}_{x_2} = \frac{0.80(1.00)}{\nu} = 50000,$$

below the transition range discussed above. The whole calculation stays in the laminar region. A measured halving distance beyond transition would need the appropriate changing flow model.

Answer

The local coefficient falls to half at $x_2 = 1.00\ \mathrm{m}$. The average coefficient over that length is $12.0\ \mathrm{W\,m^{-2}\,K^{-1}}$. The Reynolds number is 50000, consistent with laminar flow.

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